Cost Effective Traffic Grooming in WDM Rings *

Ori Gerstel and Rajiv Ramaswami
Tellabs Operations Inc., Optical Network Group, Hawthorne, NY, {ori, rajiv}@tellabs.com

Galen Sasaki
U. Hawaii, Dept. Elec. Engg., Honolulu, HI, sasaki@spectra.eng.hawaii.edu

Abstract

We provide network designs for *optical wavelength division multiplexed* (OWDM) rings that minimize overall network cost, rather than just the number of wavelengths needed. The network cost includes the cost of the transceivers required at the nodes as well as the number of wavelengths. The transceiver cost includes the cost of terminating equipment as well as higher-layer electronic processing equipment, and in practice, can dominate over the cost of the number of wavelengths in the network. The networks support dynamic (time varying) traffic streams that are at lower rates (e.g., OC-3, 155 Mb/s) than the lightpath capacities (e.g., OC-48, 2.5 Gb/s). A simple OWDM ring is the *point-to-point* ring, where traffic is transported on WDM links optically, but switched through nodes electronically. Although the network is efficient in using link bandwidth, it has high electronic and opto-electronic processing costs. Two OWDM ring networks are given that have similar performance but are less expensive. Two other OWDM ring networks are considered that are *nonblocking*, where one has a *wide sense nonblocking* property and the other has a *rearrangeably nonblocking* property. All the networks are compared using the cost criteria of number of wavelengths and number of transceivers.

1 Introduction

An *optical wavelength-division-multiplexed* (WDM) ring network (OWDM ring in short), shown in Figure 1, consists of N nodes labeled $0, 1, ..., N-1$ in the clockwise direction, interconnected by fiber links. Each link carries high-rate traffic on optical signals at many wavelengths. The network has a fixed set of wavelengths for all links which we denote by $\{\omega_0, \omega_1, ..., \omega_{W-1}\}$, where W denotes the number of wavelengths. OWDM ring networks are being developed as part of test-beds and commercial products, and are expected to be an integral part of telecommunication backbone networks. Although mesh topology WDM networks will be of greater importance in the future, at least in the near term, ring topologies are viable because SONET/SDH self-healing architectures are ring oriented.

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OWDM rings support *lightpaths*, which are all-optical communication connections that span one or more links. We will consider networks where each lightpath is full duplex, and its signals in the forward and reverse direction use the same wavelength and route. Since each lightpath is full duplex, it is terminated by a pair of *transceivers*. Here, a transceiver is generic for such systems as *line terminating equipment* (LTE) and *add/drop multiplexers* (ADM) (or more accurately, half an ADM). All lightpaths have the same transmission capacity, e.g., OC-48 (2.5Gb/s) rates.

A node in a OWDM ring is shown in Figure 2. Note that some of the lightpaths pass through the node in optical form. They carry traffic not intended for the node. The remaining lighpaths are terminated at the node by transceivers, and their traffic is converted to electronic form, and processed electronically. The electronic processing (and switching) includes systems such as ADMs and *digital crossconnect systems* (DCSs), that cross connects traffic streams. In the figure, the DCS is shown representing all the electronic processing, and the transceivers are located at the interface of the DCS and lightpaths. Now some of the received traffic may be intended for the node, in which case it is switched to a local entity through local access ports. The rest of the traffic is forwarded on other lightpaths via the transceivers. In our model, the cost of transceivers is a dominant cost.

A special case of an OWDM ring network is the *point-to-point* WDM ring network (PPWDM ring in short) shown in Figure 3. Here, each link in the network has one-hop lightpaths on each of its wavelengths. The network is called a point-to-point ring because each lightpath implements a point-to-point connection between neighboring nodes. For the network, each node has a single DCS that cross connects traffic from all the lightpaths. The DCS is *wide sense nonblocking*, which means that a traffic stream may be routed through it without disturbing existing traffic streams. Note that this network does not have a true optical node because lightpaths do not pass through nodes, i.e., traffic at each node is processed electronically.

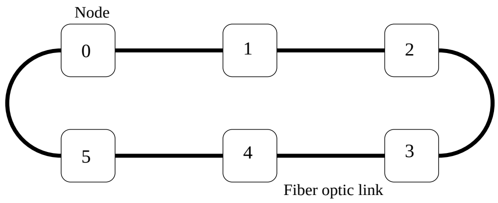

Figure 1: An optical WDM ring.

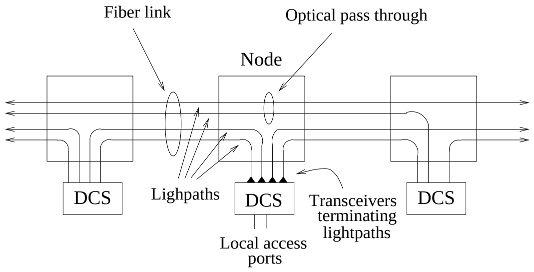

Figure 2: An optical node.

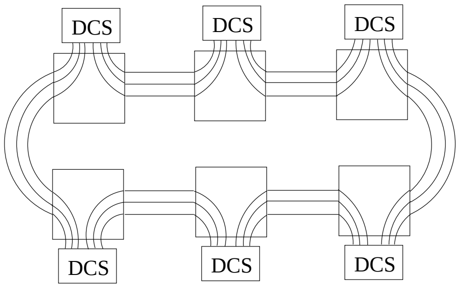

Figure 3: A point-to-point OWDM ring with three wavelengths.

The PPWDM ring has the advantage of being able to efficiently use the link bandwidth for time varying traffic. The network can route a traffic stream through it without disturbing other traffic streams as long as there is enough spare capacity along each link of the route. Hence, it will tend to be wavelength efficient. Its disadvantage is that its nodes do not have optical pass through, resulting in maximum transceiver cost. For instance, in a typical carrier network, each link may have 16 wavelengths, each carrying OC-48 data. Suppose an OWDM ring node needs to terminate only one lightpath worth of traffic. In this case, the node would ideally pass through the remaining 15 lightpaths in optical form without "processing" them. On the other hand, a PPWDM ring would require the traffic from all 16 wavelengths to be received, possibly switched through an electronic DCS, and retransmitted.

In practice however, the situation is somewhat more complicated. Each lightpath typically carries many multiplexed lower-speed traffic streams (e.g., OC-3 streams, which are at 155 Mb/s). An OWDM ring node cannot extract an individual lower-speed stream from a wavelength without first receiving the entire wavelength. Thus, in the example above, if we had to extract an individual OC-3 stream from each of the 16 wavelengths at a node, and all the remaining traffic were not intended for that node, all 16 wavelengths must be received. Note that the problem of designing networks that efficiently *grooms traffic* (i.e., multiplex/demultiplex lower-speed traffic streams onto and off-of higher capacity lighpaths) is nontrivial, and its solution can have a great impact on network cost.

In this paper we will address the problem of designing OWDM rings for cost effective traffic grooming. Our approach will be to propose and analyze a collection of OWDM ring networks under the following assumptions and criteria:

- Network costs will be dealt with explicitly. The costs of interest are (i) transceiver costs, (ii) numbers of wavelengths, and (iii) maximum numbers of hops for a lightpath. While most of the previous work on WDM networks dealt with minimizing the number of wavelengths, this paper is the first to consider transceiver costs. As it turns out, transceiver cost may reflect actual costs better than the number of wavelengths. In addition, our cost analysis give formulas that quantitatively relate network resources with traffic parameters.

- The networks have fixed lightpaths, although their placement may be optimized at start up. This is a reasonable assumption for practical WDM networks at least in the near term because (i) the traffic in a lightpath is an aggregation of many traffic streams, making it less likely to fluctuate significantly; and (ii) automatic network switching for lightpaths is not yet cost effective.

- The networks support lower-speed, full-duplex, and circuit-switched traffic streams. For example, the lightpaths may be at the OC-48 rate and support OC-3 circuit switched connections. In addition, three types of traffic models will be considered: *static*, *dynamic*, and *incremental*. *Static* traffic means that lower-speed traffic streams are set up all at once, at some initial time, and fixed thereafter. *Dynamic* traffic means that traffic streams are set up and terminated at arbitrary times. *Incremental* traffic is dynamic traffic but where traffic streams never terminate. This models the situation when traffic streams are expected to have a long holding times, as is usually the case with provisioning of high-speed connections today.

The overall network design problem comprises of two phases: first the lower-speed traffic must be aggregated on to lightpaths, so as to minimize transceiver costs as well as wavelength costs. This is the focus of our paper. The second phase may incorporate constraints in organizing the lightpaths. For instance, an OWDM network may be called upon to realize multiple SONET rings. This phase of network design is treated in a follow-on paper that also includes transceiver (ADM) costs [8]. Here, an OWDM network must realize multiple SONET rings (one ring per wavelength). However, the lightpaths are already assumed to be given and the focus is on arranging them in rings. Besides [8], we are not aware of other network design studies focusing on transceiver cost. Typically, researchers have concentrated on numbers of wavelengths, congestion, delay, or probability of blocking. We should mention that there is previous work on WDM network design for lower-speed traffic streams [2, 4, 7, 13, 14], but assuming traffic is *static*. There are also a number of papers on WDM networks with dynamic traffic (e.g., [3, 1, 9, 11, 12]), but assuming lightpaths are not fixed (they adapt to dynamic traffic). The study of (non-statistical) dynamic traffic and fixed lightpaths for OWDM networks seems to be unique to this paper.

We will now describe the network costs of interest and our specific traffic model. Then we will briefly describe our particular OWDM ring networks. The following are the network costs:

Number of Wavelengths W: Note that W is at least the maximum number of lightpaths that goes through any link.

Transceiver Cost Q: The cost Q is defined to be the average number of transceivers per node in the network. For example, for the PPWDM ring, $Q = 2W$. Note that Q is just twice the average number of lightpaths per node because a pair of transceivers terminate each lightpath.

Maximum Number of Hops $\mathcal{H}$: The cost $\mathcal{H}$ is defined to be the maximum number of hops of a lightpath. For example, for the PPWDM ring, $\mathcal{H} = 1$. It is desirable to minimize $\mathcal{H}$ since it leads to simpler physical layer designs.

Another cost to consider is the DCS cost. We will ignore it in this paper for the following reason. Our model of DCS cost has interface-ports rather than switch-fabric dominating cost. Thus, the DCS cost is proportional to the number of ports, which in turn is roughly proportional to the number of transceivers. Thus, we will "lump" the DCS cost in with the transceiver cost.

Since we ignore the DCS cost, we will assume that each node has a large *wide sense nonblocking* DCS capable of switching all the traffic through it. This assumption is realistic for practical systems and will simplify our subsequent discussion.

Now we will describe our traffic model. We will assume that our ring networks support lower-speed, circuit-switched, and full-duplexed traffic streams, all having the same rate, e.g., OC-3. The number of traffic streams that can be supported by a lightpath is assumed to be some integer denoted by c which is divisible by four. For example, if the traffic streams are OC-3 and the lightpaths are OC-48 then $c = 16$.

The traffic pattern is represented by a traffic matrix $T = [T(i,j)]$ where $T(i,j)$ denotes the number of lightpaths of traffic between nodes i and j. Thus, $c \cdot T(i,j)$ traffic streams are between i and j. Note that $T(i,j)$ can be fractional. For example, if 24 OC-3 connections (1 OC-48 = 16 OC-3s) are to be supported between i and j then $T(i,j) = 1.5$.

We will assume that there are constraints on the number of traffic streams that may terminate at nodes. In particular we will assume that for each node $i = 0, 1, ..., N-1$, there is an integer $t(i)$, which is the maximum amount of lightpath traffic that the node may terminate, i.e., at all times $t(i) \geq \sum_{j=0}^{N-1} T(i,j)$ and $t(i) \geq \sum_{j=0}^{N-1} T(j,i)$. Thus, node i can terminate $c \cdot t(i)$ traffic streams. Note that if $t(i)$ is small then it makes sense for more lightpaths to pass through node i.

The following is a list of the OWDM rings we will consider. In Section 2 we will provide a more detailed description of the networks and their costs.

PPWDM Ring: This is the PPWDM ring described earlier.

Fully-Optical Ring: For this network, between each pair of nodes i and j there are $\lceil T(i,j) \rceil$ lightpaths between them. Traffic streams between the nodes are carried directly by these connecting lightpaths. We consider this network because it has no electronic traffic grooming (which is why it is called "fully-optical"). It is therefore the opposite of the PPWDM ring which has maximal traffic grooming capability. Note that it is well suited for *static traffic* if the traffic is high enough to fill the lightpaths.

Single-Hub: This network has a node designated as a *hub*, which has lightpaths directly connecting it to all other nodes. It is *wide sense nonblocking*, i.e., traffic streams may be added without disturbing existing ones.

Double-Hub: This network has two hubs, which have lightpaths connecting them to all other nodes. This network is *rearrangeably nonblocking*, which means that it can support dynamic traffic, but it may have to rearrange existing traffic streams to make way for new ones. Note that rearranging existing traffic streams is undesirable in practical networks. However, the double-hub network is reasonably efficient in W and Q, so it could be used for static traffic.

The next two networks *perform* as well as a PPWDM ring with some number of wavelengths Λ. We are interested in such performance since a PPWDM ring is the most efficient in utilizing wavelengths. To be more precise about how these networks perform, note that the PPWDM ring has the property that it can route a traffic stream through it without disturbing existing traffic streams if the amount of traffic in each link along the route is strictly less than Λ, i.e., there is spare capacity along the route. We refer to a network having this property as being *equivalent* to a PPWDM ring network with Λ wavelengths. The next two ring networks have this property.

Hierarchical Ring: This is a simple network composed of two PPWDM subrings, and it is *equivalent* to a single PPWDM ring network with Λ wavelengths for dynamic traffic. The hierarchical ring uses more wavelengths but often uses less transceivers than the single PPWDM ring.

Incremental Ring: This a ring network that is recursively defined (or built) from smaller sections of the ring. It is *equivalent* to a PPWDM ring with Λ wavelengths for incremental traffic. It uses the same number of wavelengths and less number of transceivers than the single PPWDM ring. Note that it can also be used for static nonuniform traffic.

In Section 3, the networks will be compared using the costs W, Q, and $\mathcal{H}$, and under the *static uniform traffic.* This traffic is parameterized by a constant τ, and its pattern is

$$T(i,j) = \begin{cases} \frac{\tau}{N-1} & \text{if } i \neq j \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

It requires good network connectivity, and its uniformity simplifies analysis. It is commonly used to compare networks in the theoretical literature, and it is a traffic that can be supported by all the OWDM ring networks we will consider. Our conclusions are given in Section 4.

For the remainder of this introduction, we will present simple lower bounds for Q and W for the static uniform traffic with parameter τ. The lower bound for Q is the trivial one:

$$Q \geq \tau. \tag{2}$$

The lower bound for W is slightly more complicated to compute. Let $\mathcal{H}_{Phy}^{T}$ denote the minimum possible average number of hops to route traffic from its source to its destination.

For uniform static traffic, we can calculate this value to be

$$\mathcal{H}_{Phy}^{T} = \begin{cases} \frac{N+1}{4} & N \text{ odd}, \\ \frac{N+1}{4} + \frac{1}{4(N-1)} & N \text{ even}. \end{cases}$$

So the amount of traffic going through a link is

$$\begin{aligned} L &\geq \frac{\mathcal{H}_{Phy}^{T} \times \text{ Total traffic}}{\text{Number of links}} = \frac{\mathcal{H}_{Phy}^{T} \times \frac{1}{2}\sum_i \sum_j T(i,j)}{N} \\ &= \begin{cases} \frac{N+1}{8}\tau, & N \text{ odd}, \\ \left(\frac{N+1}{8} + \frac{1}{8(N-1)}\right)\tau & N \text{ even}. \end{cases} \end{aligned}$$

Therefore,

$$W \geq \begin{cases} \frac{N+1}{8}\tau, & N \text{ odd}, \\ \left(\frac{N+1}{8} + \frac{1}{8(N-1)}\right)\tau & N \text{ even}. \end{cases} \tag{3}$$

(Note that for non-uniform static traffic $\mathcal{H}_{Phy}^{T}$ must be replaced by the traffic-weighted average number of hops as

$$\mathcal{H}_{Phy}^{T} = \frac{\sum_{i,j} T(i,j)h(i,j)}{\sum_{i,j} T(i,j)},$$

where $h(i,j)$ is the average number of hops from i to j.)

2 Optical WDM Ring Architectures

2.1 Point-to-Point WDM Ring

Consider the PPWDM ring network and static uniform traffic as before. Assuming all traffic is routed along the shortest path in the ring, the amount of lightpath traffic on each link is

$$L = \begin{cases} \frac{1}{8}(N+1)\tau & N \text{ odd} \\ \frac{1}{8}(N+1+\frac{1}{N-1})\tau & N \text{ even} \end{cases}$$

In this case,

$$W = \begin{cases} \left\lceil \frac{1}{8}(N+1)\tau \right\rceil & N \text{ odd} \\ \left\lceil \frac{1}{8}(N+1+\frac{1}{N-1})\tau \right\rceil & N \text{ even} \end{cases}$$

Also recall that the number of transceivers per node is

$$Q = 2W,$$

and the maximum hop length

$$\mathcal{H} = 1.$$

2.2 Fully-Optical Ring

Consider a network where traffic must be routed on a single lightpath from its source to its destination. This will require setting up lightpaths between each source and destination node between which there is any traffic. This type of a network has been considered in [6] for the case of the static uniform traffic with $\tau = N-1$.

Consider the case $\tau = N-1$ and uniform static traffic. Now we need to set up one lightpath between each pair of nodes. The wavelength assignment will be done on a recursive basis as shown below. Let N be even.

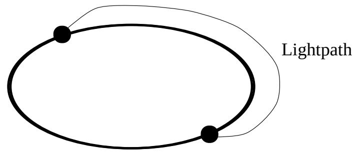

Figure 4: Setting up a lightpath between the first two nodes.

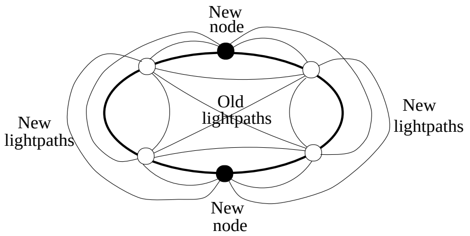

Figure 5: Setting up the lightpaths for two new nodes.

1. Start with 2 nodes on the ring (see Figure 4.) The sole lightpath that needs be set up will require 1 wavelength.
2. (Recursive step) Let k denote the number of nodes in the ring currently. While $k \leq N-2$, add 2 more nodes to the ring such that they are diametrically opposite to each other, i.e., separated by the maximum possible number of hops (see Figure 5). The two new nodes divide the ring in half, where each half has $\frac{k}{2}$ old nodes. In one half, each old node sets up a lightpath to each new node. This requires one wavelength per old node since each old node can fit its two lightpaths in a wavelength (since the lightpaths use disjoint routes). Thus, a total of $\frac{k}{2}$ new wavelengths are required. The old nodes in the other half of the ring can do the same thing and use the same $\frac{k}{2}$ wavelengths. Finally, the two new nodes require an additional wavelength to set up a lightpath between them. Thus, we need to add a total of $(k/2)+1$ new wavelengths.

So the number of wavelengths needed to do the assignment is

$$W = 1+2+3+...+\frac{N}{2} = \frac{N^2}{8} + \frac{N}{4}.$$

For arbitrary τ the wavelength assignment can be done with

$$W = \left\lceil \frac{\tau}{N-1} \right\rceil \left(\frac{N^2}{8} + \frac{N}{4}\right)$$

wavelengths, where N is even.

When N is odd, we start the procedure above with 3 nodes and add two nodes each time. The number of wavelengths in this case can be calculated to be

$$W = \left\lceil \frac{\tau}{N-1} \right\rceil \frac{N^2-1}{8}.$$

Clearly, the number of transceivers required per node is given by

$$Q = \left\lceil \frac{\tau}{N-1} \right\rceil (N-1).$$

The maximum hop length is

$$\mathcal{H} = \left\lfloor \frac{N}{2} \right\rfloor. \tag{4}$$

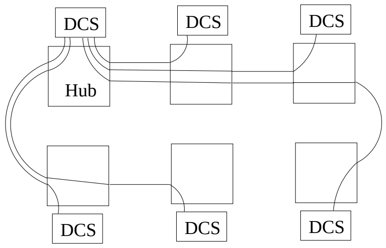

Figure 6: A single hub network for the case when $t(i) = 1$ for all nodes i.

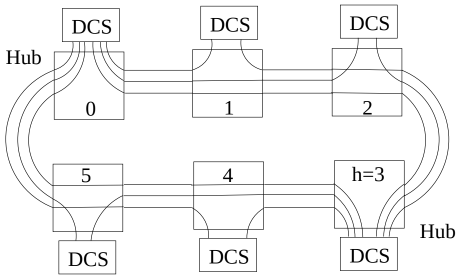

Figure 7: A double-hub network when $t(i) = 2$ for all nodes i.

2.3 Single-Hub Ring

For the *single-hub* ring network there is a node designated as the *hub*. An example of a single-hub network is shown in Figure 6. The hub node is chosen such that it achieves the maximum $\max_{0 \le i < N} t(i)$. As we shall see, this choice for the hub minimizes the number of wavelengths required. For simplicity, we will denote $\max_{0 \le i < N} t(i)$ by $t_{\max}$.

For now, without loss of generality, assume that the hub is node 0. All other nodes are connected by lightpaths directly to the hub. Thus, each node i must have $t(i)$ lightpaths to the hub. Traffic streams are routed between nodes by going through the hub. Since enough lightpaths have been provisioned between each node and the hub, the network is wide-sense nonblocking. To see this, suppose there is a pair of nodes i and j such that the amount of terminating traffic at nodes i and j is less than $t(i)$ and $t(j)$, respectively. Then there must be spare capacity on some lightpaths between node i and the hub, and between node j and the hub. Thus, a new lower speed traffic stream may be set up between nodes i and j without disturbing existing streams by using the spare capacity and going through the hub.

The number of wavelengths required is $\left\lceil \frac{1}{2} \sum_{i=1}^{N-1} t(i) \right\rceil$ because there are $\sum_{i=1}^{N-1} t(i)$ lightpaths, and we can fit two lightpath connections into a wavelength (the lightpaths on the same wavelength use disjoint routes along on the ring).

We have the following properties of the single hub ring:

- $W = \left\lceil \frac{1}{2} [\sum_{i=0}^{N-1} t(i) - t_{\max}] \right\rceil$
- $Q = 2\frac{\sum_{i=0}^{N-1} t(i) - t_{\max}}{N}$ since there are $\sum_{i=0}^{N-1} t(i) - t_{\max}$ lightpaths.
- $\mathcal{H} = N - 1$ since lightpath routes may be forced to circumvent the ring to minimize wavelengths.

For the special case $t(i) = \tau$ for all nodes i, we have the following:

- $W = \left\lceil \frac{\tau(N-1)}{2} \right\rceil$.
- $Q = 2\tau(1 - \frac{1}{N})$.
- $\mathcal{H} = \lfloor \frac{N}{2} \rfloor$ since we can arrange the lightpaths to take shortest hop paths.

Now note that since the single-hub ring is *wide sense nonblocking*, it is also rearrangeably nonblocking. The following theorem gives a lower bound on the number of wavelengths required for such a OWDM ring. Notice that the number of wavelengths for the single-hub ring is about twice as much the lower bound. However, in the next subsection, a rearrangeably nonblocking OWDM ring is given that almost meets the lower bound.

Theorem 1 *Consider a rearrangeably nonblocking OWDM ring network. Suppose N is even, and for each node $i = 0, 1, ..., N-1$, $t(i) = \tau$, where τ is integer. Then the number of wavelengths W is at least $\lceil \tau \frac{N}{4} \rceil$.*

Proof. Consider the case where for $i = 0, 1, ..., \frac{N}{2} - 1$, there is τ amount of lightpath traffic between nodes i to $i + \frac{N}{2}$. Note that the traffic must traverse $\frac{N}{2}$ links. Thus, the traffic contributed by a pair of nodes over all links in the ring is $\frac{N\tau}{2}$. Since there are $\frac{N}{2}$ pairs of nodes, the total traffic over all links in the ring is $\frac{N^2\tau}{4}$. Since there are N links, there must be at least one link with at least $\frac{N\tau}{4}$ amount of lightpath traffic. Thus, the theorem is implied. □

2.4 Double-Hub Ring

For the *double-hub* ring network, two nodes are *hubs*. An example of a double-hub ring is shown in Figure 7. Without loss of generality, assume one of the hubs is node 0, and denote the other hub by h. Each node i has communication connections to each hub, and the aggregate capacity to each hub is equivalent to $\frac{t(i)}{2}$ lightpaths. This allows node i to send (and sink) up to $c\frac{t(i)}{2}$ traffic streams to (and from) each hub.

We will now describe how the communication connections are realized by lightpaths. We will use the following terminology and definitions. The nodes $0, 1, 2, ..., h-1$ will be referred to as *side 1* of the ring. The rest of the nodes $h, h+1, ..., N-1$ will be referred to as *side 2* of the ring. We will also use the notation $rem(t(i)/2)$ to denote the remainder of $\frac{t(i)}{2}$. Note that $rem(t(i)/2)$ is zero if $t(i)$ is even and $\frac{1}{2}$ if $t(i)$ is odd. We will refer to nodes that have $rem(t(i)/2) = \frac{1}{2}$ as *odd traffic* nodes.

We will now describe how nodes in side 1 connect to the hubs. (Note that the nodes in side 2 are connected to the hubs in a similar way.) Each node i in side 1 uses $\left\lfloor \frac{t(i)}{2} \right\rfloor$ wavelengths to carry $\left\lfloor \frac{t(i)}{2} \right\rfloor$ lightpaths directly to each hub. The lightpaths are routed only using links on side 1 of the ring. Note that it is possible to use only $\left\lfloor \frac{t(i)}{2} \right\rfloor$ wavelengths because lightpaths going to different hubs have disjoint routes.

Note that if $t(i)$ is odd then node i must have an additional $\frac{1}{2}$ $(= rem(t(i)/2))$ worth of lightpath connection to each hub. These "half-a-lightpath" connections are realized by having two odd-traffic nodes share a wavelength. For example, if u and v are odd-traffic nodes sharing a wavelength and $u < v$ then there would be lightpaths between the pairs $(0, u)$, (u, v), and (v, h). Thus, if $u \neq 0$ then there would be three lightpaths, and if $u = 0$ then there would be two lightpaths. Now nodes u and v can use half the bandwidth of a lightpath to carry $\frac{c}{2}$ traffic streams to and from each hub.

It is straight forward to check that number of wavelengths required for side 1 of the ring is $\left\lceil \frac{1}{2} \sum_{i=0}^{h-1} t(i) \right\rceil$. It is also straight forward to check that the number of lightpaths for side 1 is at most $\sum_{i=1}^{h-1} t(i) + \left\lfloor \frac{t(0)}{2} \right\rfloor + \left\lceil \sum_{i=0}^{h-1} rem(t(i)/2) \right\rceil$. A similar calculation can be done for side 2. Thus, we have

- $W = \max\left\{ \left\lceil \frac{1}{2} \sum_{i=0}^{h-1} t(i) \right\rceil, \left\lceil \frac{1}{2} \sum_{i=h}^{N-1} t(i) \right\rceil \right\}$,
-
$$Q \leq \frac{2}{N}\left(\sum_{i=0}^{N-1} t(i) + \left\lceil \sum_{i=0}^{h-1} rem(t(i)/2) \right\rceil + \left\lceil \sum_{i=h}^{N-1} rem(t(i)/2) \right\rceil \right)$$
- $\mathcal{H} = N - 1$

For the special case of $h = \lceil N/2 \rceil$ and for all nodes i, $t(i) = \tau$, we have the following.

- $W = \lceil \frac{\tau}{2} \lceil \frac{N}{2} \rceil \rceil$.
- $Q \leq 2\left(\tau + rem(\tau/2) + \frac{1}{N}\right) \leq 2\tau + 1 + \frac{1}{N}$.
- $\mathcal{H} = \lceil \frac{N}{2} \rceil$.

Theorem 2 *The double-hub ring network is rearrangeably nonblocking.*

Proof. The double-hub ring can be viewed as a switching network where lower-speed traffic streams are routed between nodes via hub nodes. Note that the traffic streams are full duplex (i.e., bidirectional) so they do not have distinct source and destination nodes typically used to define connections in switching networks. We will artificially give each traffic stream a *direction*, so that it will have a source and destination. Note that the directions are used for routing purposes only, and the traffic streams are still full duplex. Also note that the directions for traffic streams may change over time which may be necessary for rerouting.

We can assume that for any collection of traffic streams, there are directions for them so that at each node i, at most $c\frac{t(i)}{2}$ streams are directed into it or out of it. This assignment can be done as follows. Since each node i has an even value for $c \cdot t(i)$, we may assume that each node i terminates exactly $c \cdot t(i)$ traffic streams. Otherwise, dummy streams can be added until it is true. Since there are an even number of traffic streams incident to any node, we can find an *Euler tour* where the streams are treated as *edges* in a *multigraph*. The traversal of such a tour gives directions to the streams such that at each node i, exactly $c\frac{t(i)}{2}$ (real or dummy) streams are directed into and out of it.

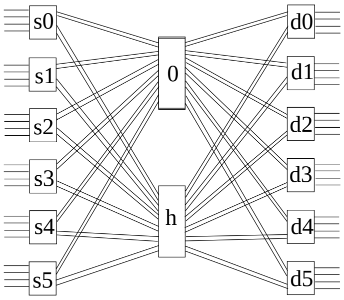

Figure 8: A three stage switch for $N = 6$, $c = 4$, and $t(i) = 2$ for all nodes i.

With the traffic streams directed, the double-hub ring can be viewed as emulating a three stage switching network, as shown in Figure 8, that supports directed traffic streams. The first stage has N vertices denoted by $s_0, s_1, ..., s_{N-1}$, where s_i represents node i in the ring network. The second stage has two vertices representing the two the hubs. The third stage has N vertices denoted by $d_0, d_1, ..., d_{N-1}$, where d_i also represents node i in the ring network. Hence, node i in the ring is represented by two vertices s_i and d_i in the three stage switching network.

Each vertex s_i in the first stage has $c\frac{t(i)}{2}$ input links which represents the fact that node i in the ring can source $c\frac{t(i)}{2}$ directed traffic streams. Similarly, each vertex d_i in the third stage has $c\frac{t(i)}{2}$ output links which represents the fact that node i in the ring can be the destination of $c\frac{t(i)}{2}$ directed traffic streams.

Each vertex s_i in the first stage has $c\frac{t(i)}{4}$ links to each vertex in the second stage, and each vertex d_i in the third stage has $c\frac{t(i)}{4}$ links from each vertex in the second stage. Thus, vertices s_i and d_i have a total of $c\frac{t(i)}{2}$ links to each hub. These links represent the fact that node i in the ring network can have $c\frac{t(i)}{2}$ traffic streams to each hub.

The three stage switching network is rearrangeably nonblocking. This can be shown by first transforming it into a three stage Clos network (see [10] for a description of a Clos network). In particular, each vertex s_i in the first stage is transformed into $c\frac{t(i)}{2}$ vertices, each having two input links and one link to each second stage vertex. Similarly, each vertex d_i in the third stage is transformed into $c\frac{t(i)}{2}$ vertices, each having two output links, and one link from each second stage vertex. The Clos network is rearrangeably nonblocking because there are two input links at each first stage vertex, two output links at each third stage vertex, and two vertices in the second stage [15, 5]. The original three stage network is rearrangeably nonblocking because it can emulate the Clos network. Hence, the double hub ring is rearrangeably nonblocking. □

2.5 Hierarchical Ring

In this section we will describe an OWDM ring network that is *equivalent* to a PPWDM ring with Λ wavelengths for dynamic traffic, where Λ is some integer. We will refer to it as the *hierarchical* ring. To simplify our discussion, we will assume that for each node i, $t(i) = \tau$, where τ is an integer.

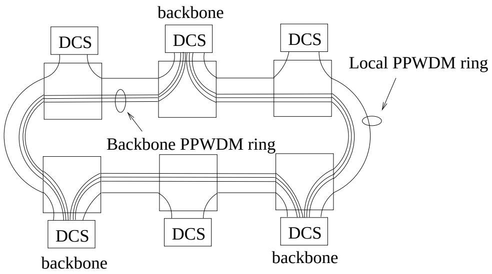

Figure 9: A hierarchical ring with parameter $\alpha = 2$, $\tau = 1$, and $\Lambda = 3$.

Figure 9 shows an example hierarchical ring.

The hierarchical ring is parameterized by an integer $\alpha > 0$. The ring has a total of $W = \Lambda + \tau(\alpha - 1)$ wavelengths. Λ of the wavelengths are referred to as *backbone* wavelengths, and the other $\tau(\alpha - 1)$ wavelengths are referred to as *local* wavelengths. The nodes of the ring are also classified into two types: *local* and *backbone*. The nodes are arranged such that there are at most $\alpha - 1$ local nodes between any consecutive pair of backbone nodes. In particular, the backbone nodes are 0, α, 2α,..., $(\lceil \frac{N}{\alpha} \rceil - 1)\alpha$, while the other nodes are local nodes. The backbone wavelengths are used to form a PPWDM ring among the backbone nodes. In other words, lightpaths are formed between consecutive backbone nodes using the backbone wavelengths. We will refer to this as the *backbone PPWDM ring*. The local wavelengths are used to form a PPWDM ring among all nodes. In other words, lightpaths are formed between consecutive nodes using local wavelengths. We will refer to this as the *local PPWDM ring*.

Note that the hierarchical ring is a hierarchical network composed of the a backbone PPWDM ring and the local PPWDM ring. The local PPWDM ring is used to route traffic between local nodes and backbone nodes; and the backbone PPWDM ring is used to route traffic across the ring between backbone nodes. For example, a traffic stream will be routed between its terminating nodes as follows. From one of its terminating nodes, it will go along a local wavelength until it reaches the first backbone node. Then it will go along the backbone PPWDM ring until it gets to the last backbone node before its other terminating node. Then it is routed along a local wavelength until it reaches its other terminating node. Since the local PPWDM ring has enough wavelengths, traffic streams can always be routed along it to get access to the next backbone node without disturbing existing streams. In addition, since the backbone PPWDM has Λ wavelengths, the hierarchical ring is equivalent to a PPWDM network with Λ wavelengths for dynamic traffic.

It is straight forward to show the following properties of the hierarchical ring.

- $W = \Lambda + (\alpha - 1)\tau$.
- $Q = 2(\alpha - 1)\tau + 2\frac{\Lambda}{N} \lceil \frac{N}{\alpha} \rceil$.
- $\mathcal{H} = \alpha$.

Note that the value of α can be chosen to minimize Q. For large N, the optimal value of α is approximately $\sqrt{\Lambda/\tau}$.

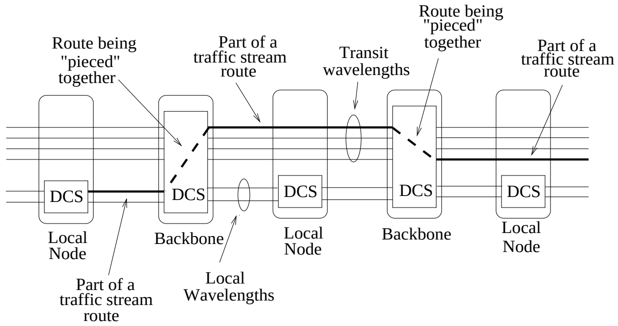

Figure 10: A section of 5 nodes of the incremental ring.

2.6 Incremental Ring

In this section we will describe an OWDM ring network that is similar to the hierarchical ring in the previous subsection. We will call it the *incremental* ring network because it is *equivalent* to a PPWDM ring with the same number of wavelengths for *incremental* traffic. To make the discussion simpler, we will assume that the traffic is incremental. We will also assume that $N = 2^k$ for some integer k, and that for all nodes i, $t(i) = \tau$ for some integer τ.

We will give a simple example of the incremental ring to illustrate it. Consider an OWDM ring with an even number of nodes and Λ wavelengths, where Λ is some integer. Figure 10 shows a section of five nodes of the ring. The even numbered nodes are called *backbone* nodes, and the odd numbered nodes are called *local nodes*. At backbone nodes, all wavelengths are terminated by transceivers. At local nodes, the wavelengths $\{\omega_0, ..., \omega_{\tau-1}\}$ are terminated by transceivers. The wavelengths $\{\omega_0, ..., \omega_{\tau-1}\}$ will be called the *local* wavelengths, and the wavelengths $\{\omega_\tau, ..., \omega_{\Lambda-1}\}$ will be called *transit* wavelengths.

The effect of having backbone nodes terminate every wavelength is that parts of routes for a traffic stream may be "pieced" together at backbone nodes through DCSs. So the problem of routing a traffic stream through the ring reduces to a problem of routing a traffic stream just between consecutive backbone nodes.

So let us look at a segment of the ring between two backbone nodes. In particular, consider the segment consisting of a local node u and its two neighboring backbone nodes b_1 and b_2. Suppose a traffic stream is to be routed through it. Call the traffic stream *local* if it terminates at u, and call it *transit* otherwise. The routing algorithm for the segment is as follows. If the traffic stream is transit then route it along the transit wavelengths, and if the traffic stream is local then route it along the local wavelengths to terminate at u.

This algorithm will always route the traffic stream without disturbing existing ones, as long as there is spare capacity along the route. We will verify this. First notice that while the transit wavelengths are unfilled, transit traffic are only in these wavelengths. Also, since node u can terminate at most τ amount of traffic, the local traffic are only in the local wavelengths. Thus, during this time new traffic streams can be added without disturbing existing ones because there is spare capacity in transit wavelengths for transit traffic and there is spare capacity in local wavelengths for local traffic. Now at some point, the transit traffic may fill the transit wavelengths. However, then the only spare capacity in all

	W	Q	$\mathcal{H}$
Lower bounds	Λ	τ	1
PPWDM	Λ	2Λ	1
Fully Optical	$\left\lceil \frac{\tau}{N-1} \right\rceil \times \left(\frac{N^2}{2} + \frac{N}{4} \right)$	$\left\lceil \frac{\tau}{N-1} \right\rceil \times (N-1)$	$\left\lfloor \frac{N}{2} \right\rfloor$
Single-Hub	$\left\lceil \frac{\tau(N-1)}{2} \right\rceil$	$2\tau \left(1 - \frac{1}{N}\right)$	$\left\lfloor \frac{N}{2} \right\rfloor$
Double-Hub	$\left\lceil \tau \frac{N}{4} \right\rceil$	$2\left(\tau + rem(\tau/2) + \frac{1}{N}\right)$	$\left\lceil \frac{N}{2} \right\rceil$
Hier-archical	$\Lambda + (\alpha - 1)\tau$	$2\left((\alpha-1)\tau + \frac{\Lambda}{N}\left\lceil \frac{N}{\alpha} \right\rceil\right)$	α
Incremental	Λ	$(\leq) 2\tau \log_2 N$	$(\leq) \frac{\Lambda}{\tau} + 1$

Table 1: A comparison of different OWDM ring architectures for the static uniform traffic. N is assumed to be even, and $\Lambda = \left\lceil \frac{\tau}{8}\left(N + 1 + \frac{1}{N-1}\right)\right\rceil$.

the wavelengths is in the local wavelengths. Since the local wavelengths are terminated by transceivers by all nodes, a traffic stream may be routed through them without disturbing existing streams as long as there is spare capacity.

This means that the incremental ring will support traffic that a PPWDM ring with the same number of wavelengths can support. Therefore, it is *equivalent* to a PPWDM ring with Λ wavelengths for incremental traffic.

The full description of the incremental ring is more complicated than the previous example which we omit for the sake of brevity. The following are properties of the incremental ring.

- $W = \Lambda$.
- $Q = \frac{2}{N}\left(2\Lambda + \sum_{i=1}^{k-1} 2^i \min\{\Lambda, \tau\left(\frac{N}{2^i} - 1\right)\}\right)$, which is at most $2\tau \log_2 N$ if $\Lambda \leq \tau(N-1)$.
- $\mathcal{H} = 2^J$, where $J = \lceil \log_2(\Lambda/\tau + 1)\rceil - 1$. Note that $\mathcal{H}$ has the simpler upper bound $\frac{\Lambda}{\tau} + 1$.

3 Comparisons

In this section, we will compare the OWDM ring networks from the previous section. Table 1 summarizes the key costs for the networks assuming the static uniform traffic pattern with parameter τ (i.e., $T(i,j) = \frac{\tau}{N-1}$ if $i \neq j$). Here, the value of N is assumed to be even, and the value for Λ for the hierarchical and incremental rings is assumed to be the lower bound of Inequality (3) i.e., $\Lambda = \left\lceil \frac{\tau}{8}(N + 1 + \frac{1}{N-1})\right\rceil$.

To simplify the comparison, we provide Table 2 which has the approximate costs assuming the static uniform traffic, and also assuming that $N \gg 1$ and $\tau \leq N - 1$. The costs in Table 2 are approximate because they exclude low order terms. Note that $\tau \leq N-1$ means that each pair of nodes has no more than one lightpath worth of traffic between them. Also note that Λ is approximately $\frac{1}{8}\tau N$. For the hierarchical ring, we assume that $\alpha = \sqrt{\frac{N}{8}}$ ($\approx \sqrt{\frac{\Lambda}{\tau}}$), which minimizes transceiver cost.

Based upon Table 2 we draw the following conclusions:

	Approximate Costs		
	W	Q	$\mathcal{H}$
Lower bounds	$\frac{1}{8}\tau N$	τ	1
PPWDM	$\frac{1}{8}\tau N$	$\tau \frac{N}{4}$	1
Fully Optical	$\frac{1}{8}N^2$	N	$\frac{N}{2}$
Single-Hub	$\frac{1}{2}\tau N$	2τ	$\frac{N}{2}$
Double-Hub	$\frac{1}{4}\tau N$	$2(\tau + 1)$	$\frac{N}{2}$
Hierarchical ($\alpha = \sqrt{\frac{N}{8}}$)	$\left(\frac{1}{8} + \sqrt{\frac{1}{8N}}\right)\tau N$	$\tau\sqrt{2N}$	$\sqrt{\frac{N}{8}}$
Incremental	$\frac{1}{8}\tau N$	$2\tau \log_2 N$	$\frac{N}{8}$

Table 2: Approximate costs for different OWDM ring networks assuming the static uniform traffic, N is large and $\tau \leq N - 1$.

- If wavelengths are plentiful then the single-hub ring is a good choice since it has low transceiver cost and can support dynamic traffic. The double-hub ring is a good choice if the traffic is static (and not necessarily uniform), since it requires only half the number of wavelengths and has about the same transceiver cost.
- If wavelengths are precious then the PPWDM, hierarchical, and incremental rings are reasonable choices for OWDM ring networks since they use minimal wavelengths. The PPWDM ring provides the most efficient use of wavelengths for dynamic traffic. If there are some spare wavelengths then the hierarchical ring can potentially reduce the transceiver cost. If the traffic is static (and not necessarily uniform) then the incremental ring is the best choice.

An interesting point is that the fully-optical network has the smallest transceiver cost in the range $\frac{N}{2} \leq \tau < N$. For this range, each pair of nodes has at least half a lightpath worth of traffic between them. For smaller values of τ, the single-hub ring has lower transceiver cost.

Note that Table 2 is based on the unrealistic assumption that N is very large. Table 3 shows W and Q values for a more realistic value of N, in particular $N = 8$. (Note that Table 3 may not equal the formulas in Table 1 because Table 1 includes upper bounds that are not necessarily tight.) Note that the hierarchical ring is not considered because for $N = 8$ the optimal value for α is 1 (i.e., hierarchical ring = PPWDM ring).

Let us consider the case when the number of wavelengths in the OWDM ring is 16. We will determine the smallest transceiver cost for the different values of τ. For small values of τ, in the range $\tau = 1, 2, 3$ and 4, the single hub has the smallest transceiver cost. For $\tau = 4, 5, 6$, and 7, the fully-optical ring has the smallest Q. So if wavelengths are abundant then single-hub and fully optical rings lead to the smallest transceiver cost.

Now let us consider the case when the number of wavelengths in the ring is 8, i.e., wavelengths are a little more scarce. Then the fully-optical ring can be discounted since it always requires $W = 10$. For $\tau = 1$ and 2, the single-hub ring has the smallest transceiver cost. But for larger τ, the single-hub requires more than 8 wavelengths. For $\tau = 3$ and 4, the double-hub has the smallest transceiver cost. But for

τ	Single H		Double H		Fully O		PPWDM		Incr	
	W	Q	W	Q	W	Q	W	Q	W	Q
1	4	1.75	2	2.5	10	7	2	4	2	3
2	7	3.5	4	3.5	10	7	3	6	3	5
3	11	5.25	6	6	10	7	4	8	4	7
4	14	7	8	7	10	7	5	10	5	9
5	18	8.75	10	9.5	10	7	6	12	6	11
6	21	10.5	12	10.5	10	7	7	14	7	13
7	25	12.5	14	13	10	7	8	16	8	15

Table 3: W and Q when $N = 8$ and for various values of τ.

larger τ, it requires more than 8 wavelengths. For $\tau = 5, 6$, and 7, both the PPWDM and incremental rings require at most 8 wavelengths. The incremental ring has a slight advantage in transceiver cost. Note that this example shows different architectures provide better transceiver costs over different values of τ.

4 Conclusions

We have proposed and analyzed a number of OWDM ring networks. At one extreme is the single-hub ring that requires large amounts of bandwidth (wavelengths) but has small transceiver cost. At the other extreme is the PPWDM ring that requires minimal bandwidth (wavelengths) but has maximum transceiver cost. In the middle we have the hierarchical ring that provides a trade-off between numbers of wavelengths and transceiver costs. Also in the middle, we have the double-hub and incremental rings. These last two do not support fully dynamic traffic, but seem to be reasonable solutions for static nonuniform traffic. On the theoretical side, we showed that the double-hub network is a near optimal rearrangeably nonblocking ring network.

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