Signals & Systems
HW

Problem Set 1 (Due Friday 1/30)
Problems A1.1, A1.2, 1.52, 1.53, 1.54

Problem A1.1
Determine whether or not each of the following continuous-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) \( x(t) = 3 \cos(\sqrt{2}t + \frac{\pi}{3}) \)
(b) \( x(t) = e^{j(2\pi t - 1)} \)
(c) \( x(t) = [\sin(4t - \frac{\pi}{3})]^2 \)
(d) \( x(t) = \mathcal{O}d\{\cos(4t)u(t)\} \)
(e) \( x(t) = \mathcal{O}d\{\sin(4t)u(t)\} \)
(f) \( x(t) = \sum_{n=-\infty}^{\infty} e^{-(t-n)}u(t - n) \)

Problem A1.2
Determine whether or not each of the following discrete-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) \( x[n] = \sin \left( \frac{\pi}{8}n + 1 \right) \)
(b) \( x[n] = \cos \left( \frac{\pi^2}{8}n - \pi \right) \)
(c) \( x[n] = \cos \left( \frac{\pi}{4}n^2 \right) \)
(d) \( x[n] = \cos \left( \frac{\pi}{2}n \right) \cos \left( \frac{\pi}{7}n \right) \)
(e) \( x[n] = 2 \cos \left( \frac{\pi}{8}n \right) + \sin \left( \frac{\pi}{8}n \right) - 4 \cos \left( \frac{\pi}{8}n + \frac{\pi}{8} \right) \)
Problem Set 2 (Due Friday 2/6)


Problem A1.3

Consider the following properties

1. Memoryless
2. Time-invariant
3. Linear
4. Causal
5. Stable

Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answers. In each example, \( y(t) \) denotes the system output and \( x(t) \) is the system input.

(a) \( y(t) = x(t - 2) + tx(3 - t) \)

(b) \( y(t) = \cos(-3t)x(t)^2 \)

(c) \( y(t) = \int_{-\infty}^{t} x(\tau)d\tau \)

(d) \( y(t) = \begin{cases} 0 & t < 0 \\ x(t) + x(t + 2) & t \geq 0 \end{cases} \)

(e) \( y(t) = \begin{cases} 0 & x(t + 2) < 0 \\ x(t) + x(t + 2) & x(t + 2) \geq 0 \end{cases} \)

(f) \( y(t) = x(3t) \).

(g) \( y(t) = \frac{dx(t+1)}{dt} \)

Problem A1.4

Consider the following properties

1. Memoryless
2. Time-invariant
3. Linear
4. Causal
5. Stable

Determine which of these properties hold and which do not hold for each of the following discrete-time systems. Justify your answers. In each example, \( y[n] \) denotes the system output and \( x[n] \) is the system input.

(a) \( y[n] = -nx[-n] \)

(b) \( x[n] = x[2n - 2] - 2x[n - 8] \)

(c) \( y[n] = \cos(\pi n)x[n] \)
(d) \( y[n] = \mathcal{E}\{x[n + 1]\} \)

(e) \( y[n] = \begin{cases} 
  x[n] & n \geq 1 \\
  0 & n = 0 \\
  x[n - 1] & n \leq -1 
\end{cases} \)

(f) \( y[n] = \begin{cases} 
  x[n + 1] & n \geq 1 \\
  0 & n = 0 \\
  x[n] & n \leq -1 
\end{cases} \)

(g) \( y[n] = 4x[4n - 1] \)

**Problem A1.5**

Determine if the following systems are

(a) time-invariant

(b) stable

(c) causal

(d) linear.

**Justify your answer**

1. A system with input \( x(t) \) giving output

   \[ y(t) = \int_{t-10}^{t} \cos(\tau)x(\tau)d\tau \]

2. A system with input \( x[n] \) giving output

   \[ y[n] = n^2 x[n^2] \]
Problem Set 3 (Due Friday 2/13)

Problems 1.29, A1.6, A2.1, 2.24

Problem A1.6

When the input to a linear time-invariant system is given by $x_1(t)$, the output is $y_1(t)$, see the figure.

(a) If the input is $x_2(t)$ (see the figure), what is the output?

(b) If the input is $x_3(t)$ (see the figure), what is the output?

Problem A2.1

Compute the convolution $y[n] = x[n] * h[n]$ of the following pair of signals

(a) $x[n] = \alpha^n u[n], h[n] = \beta^n u[n - 1], \alpha \neq \beta$.

(b) $x[n] = \alpha^n u[n], h[n] = \alpha^n u[n - 1]$

(c) $x[n] = \left(\frac{1}{2}\right)^n u[n - 3], h[n] = 3^n u[4 - n]$.

(d) $x[n] = \begin{cases} 1 & 0 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$, $h[n] = \begin{cases} 1 & 1 \leq n \leq 5 \\ 1 & 8 \leq n \leq 13 \\ 0 & \text{otherwise} \end{cases}$
Problem Set 4 (Due Friday 2/20)

Problems 2.22, A2.3, A2.4, A2.5

Problem A2.3

The following are the impulse responses of discrete-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers. The justification for stability needs to in terms of the summation condition.

(a) \( h[n] = \left( \frac{1}{2} \right)^n u[n] \)
(b) \( h[n] = (-1.8)^n u[n - 2] \)
(c) \( h[n] = (2)^n u[-n + 1] \)
(d) \( h[n] = \left( \frac{1}{2} \right)^n u[-1 - n] \)
(e) \( h[n] = (-\frac{1}{2})^n u[n] + 1.01^{n+1} u[n + 1] \)
(f) \( h[n] = (-\frac{1}{2})^n u[n] + 0.99^n u[1 - n] \)
(g) \( h[n] = n^2 \left( \frac{1}{2} \right)^n u[n - 3] \)

Problem A2.4

The following are the impulse responses of continuous-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers. The justification for stability needs to in terms of the integration condition.

(a) \( h(t) = e^{-4t} u(-t - 2) \)
(b) \( h(t) = e^{-\frac{1}{2}t} u(t - 2) \)
(c) \( h(t) = e^{-0.1t} u(t + 100) \)
(d) \( h(t) = e^{4t} u(t - 2) \)
(e) \( h(t) = e^{|t|} \)
(f) \( h(t) = t^2 e^{-t/2} u(t) \)
(g) \( h(t) = (2e^{-t/2} + e^{10-t})^{10} u(t + 2) \)

A2.5
Problem Set 5 (Due Friday 3/6)

Problems 2.30, 2.32, 3.22, 3.24
Problem Set 6 (Due Friday 3/13)
Problems 3.25, 3.26, 3.23, A3.5

Problem A3.5
The following plot shows a signal $x(t)$ with period $T = 4$.

1. Find the Fourier coefficients of $x(t)$.
2. Sketch the signal with period $T = 4$ having Fourier coefficients $a_k = \begin{cases} 0 & k = 0 \\ -3\sin\left(\frac{k\pi}{2}\right) & k \neq 0 \end{cases}$
3. Sketch the signal with period $T = 4$ having Fourier coefficients $a_k = \begin{cases} -\frac{1}{2} & k = 0 \\ -6e^{j\frac{k\pi}{2}} \sin\left(\frac{k\pi}{2}\right) & k \neq 0 \end{cases}$
4. Sketch the signal with period $T = 4$ having Fourier coefficients $a_k = \begin{cases} -\frac{1}{2} j\sin\left(\frac{k\pi}{2}\right) & k = 0 \\ \frac{3}{2} j\sin\left(\frac{k\pi}{2}\right) & k \neq 0 \end{cases}$

Problem Set 7 (Due Friday 3/20)
Problems 3.33, 3.34 (c), 3.35, 3.45

Problem Set 8 (due Monday 4/6)
Problems A3.6 (1), 5.21(b)(c)(d)(f)(j), 5.22(a)(b)(d)(f)

Problem A3.6
Find the Fourier coefficients of the following signals
1. The continuous time signal $x(t)$ with period $T = 4$ given by $x(t) = \begin{cases} e^t & -2 < t \leq 0 \\ 0 & 0 < t \leq 2 \end{cases}$
in the interval $-2 < t \leq 2$. 
2. The discrete-time signal $x[n]$ with period $T = 8$ given by

$$x[n] = (-1)^n$$

in the interval $0 \leq n \leq 7$.

**Problem Set 9 (due Friday 4/17)**

4.21(a)(b)(c)(e)(f), 4.22(a)(c)(e)

**Problem Set 10 (due Friday 4/24)**

4.23, 5.23, 4.24, 4.25, 5.29

**Problem Set 11 (due Friday 5/1)**

5.34, 5.33 (a)(b-i)(c-i), 4.34, 4.33, A5.1

**Problem A5.1**

Consider a discrete-time LTI system described by the difference equation

$$y[n] + \frac{5}{2} y[n - 1] - \frac{3}{2} y[n - 2] = x[n] + 3x[n - 2]$$

1. Find the frequency response of the system.

2. Find the impulse response of the system corresponding to the frequency response.

**Problem Set 12 (due Thursday 5/7)**

Problems 5.24, 4.32, A7.1, A7.2

**Problem A7.1**

A signal $x(t)$ has the Fourier transform $X(j\omega)$ indicated in the figure. The signal is sampled to obtain the discrete time signal

$$x[n] = x(nT)$$

1. Sketch the Fourier transform $X_T(j\omega)$ of $x[n]$ for $T = \frac{1}{10}$.

2. Can $x(t)$ be recovered for $T = \frac{1}{10}$? How? What is the maximum value of $T$ so that $x(t)$ can be recovered?
Problem A7.2

The signal \( x(t) \) is given by

\[
x(t) = \frac{\sin(4\pi t)}{t}
\]

(except at \( t = 0 \), where \( x(0) = 4 \)). The signal is sampled to obtain the discrete time signal

\[
x[n] = x(nT)
\]

1. What is the minimum sampling frequency that can be used to avoid aliasing?

2. Suppose the signal is sampled with a sampling frequency \( \omega_s = 6\pi \). Sketch the Fourier transform \( X_T(j\omega) \) of \( x[n] \).

3. The signal \( x[n] \) (with \( s \) from question 2) is passed through a reconstruction filter with

\[
H(j\omega) = \begin{cases} 
1 & |\omega| \leq \pi \\
0 & \text{otherwise}
\end{cases}
\]

Write down an expression for the output of the filter \( y(t) \).