EE342 Fall 2010
PS 5 solutions

#5-3

No, for both cases the function is negative for x>2. Therefore the functions cannot be pdfs.

#5-6

(a) \( E[X] = \frac{1}{4} \int_0^\infty x^2 e^{-x^2/2} dx = \frac{\sqrt{\pi}}{4} \) 
(b) By symmetry of \( f(x) \) about \( x = 0 \), \( E[X] = 0 \).
(c) \( E[X] = \int_5^\infty \frac{5}{x} dx = \infty \).

#5-11

\( X \) is uniform on \((0, L)\).
\[
P \left( \min \left( \frac{X}{L-X} \right) < \frac{1}{4} \right) = 1 - P \left( \min \left( \frac{X}{L-X} \right) > \frac{1}{4} \right)
\]
\[
= 1 - P \left( \frac{X}{L-X} > \frac{1}{4}, \frac{L-X}{X} > \frac{1}{4} \right) = 1 - P \left( X > \frac{L}{5}, X < \frac{4L}{5} \right)
\]
\[
= 1 - P \left( \frac{L}{5} < X < \frac{4L}{5} \right) = 1 - \frac{2}{5} = \frac{3}{5}
\]

#5-15

(a) \( P\{X > 5\} = 1 - \Phi \left( \frac{5 - 10}{6} \right) = \Phi \left( \frac{5}{6} \right) = .7977 \)
(b) \( P\{4 < X < 16\} = \Phi \left( \frac{16 - 10}{6} \right) - \Phi \left( \frac{4 - 10}{6} \right) = .2277 \)
(c) \( P\{X < 8\} = \Phi \left( \frac{8 - 10}{6} \right) = .3695 \)
(d) \( P\{X < 20\} = \Phi \left( \frac{20 - 10}{6} \right) = .9522 \)
(e) \( P\{X > 16\} = 1 - \Phi \left( \frac{16 - 10}{6} \right) = .1587 \)

#5-20

Let \( X \) denote the number in favor. Then \( X \) is binomial with mean 65 and standard deviation \( \sqrt{65(.35)} = 4.77 \). Also let \( Z \) be a standard normal random variable.

(a) \( P\{X \geq 50\} = P\{X \geq 49.5\} = P \left\{ \frac{X - 65}{4.77} = \frac{-15.5}{4.77} \right\} \approx P\{Z \geq -3.25\} \approx .9994 \)
(b) \( P\{59.5 \leq X \leq 70.5\} = P\{-5.5/4.77 \leq Z \leq 5.5/4.77\} = 2P\{Z \leq 1.15\} - 1 = .75 \)
(c) \( P\{X \leq 74.5\} = P\{Z \leq 9.5/4.77\} \approx .977 \)
#5-8 (Theoretical Exercise)

Since $0 \leq X \leq c$, it follows that $X^2 \leq cX$. Hence,

$$Var(X) = E[X^2] - (E[X])^2$$

$$\leq E[cX] - (E[X])^2$$

$$= cE[X] - (E[X])^2$$

$$= E[X](c - E[X])$$

$$= c^2[\alpha(1-\alpha)] \text{ where } \alpha = E[X]/c$$

$$\leq c^2/4$$

where the last inequality first uses the hypothesis that $P\{0 \leq X \leq c\} = 1$ to calculate that $0 \leq \alpha \leq 1$ and then uses calculus to show that $\max_{0 \leq \alpha \leq 1} \alpha(1-\alpha) = 1/4$.

2) a) mean is 1 and variance is 4/3.

b) sample pdf and sample CDF approximate pdf and CDF with plots shown below.

Sample mean is 1.0052 and sample variance is 1.3343.

Matlab commands:
```matlab
dt=.002; t=-2:dt:4;
pdfu=.25*(t>= -1 & t<3);
subplot(2,2,1);
plot(t,pdfu)
axis([-2 4 0 .3])
ylabel('pdf of uniform [-1,3] RV')
cdfu=cumsum(pdfu)*(dt);
subplot(2,2,2)
plot(t,cdfu)
axis([-2 4 0 1.2])
ylabel('CDF of uniform [-1,3] RV')
u=4*rand(1,100000)-1;
[histu tu]= hist(u,100);
tu = [-1 tu 3];
histu= [0 histu/4000 0];
subplot(2,2,3)
plot(tu,histu)
axis([-2 4 0 0.3])
ylabel('sample pdf')
cdfus= cumsum(histu);
subplot(2,2,4)
plot(tu,cdfus/25)
axis([-1 3 0 1.2])
ylabel('sample CDF')
```

3) a) mean is 3 and variance is 4.

b) sample pdf and sample CDF closely approximate pdf and CDF with plots shown below. Sample mean is 3.0037 and sample variance is 3.9822.

Matlab commands:
```matlab
dt=.002; t=-3:dt:9; mg=3; vg=4;
pdf= 1/sqrt(2*pi*vg)*exp(-.5*(t-3).^2/vg);
subplot(2,2,1);
```