Objective: Observe the TE modes in a rectangular wave guide and plot the E- and H-field lines.

1. Background

The electromagnetic field inside a rectangular waveguide can be determined by solving Maxwell’s equations with appropriate boundary conditions. In many cases, waveguide walls are good conductors. For convenience we assume these conductors are PEC (perfect electric conductors), i.e., their conductivity is infinitely large. The electric field inside a PEC material has thus to be zero; otherwise the current induced will be infinitely large (since \( j = \sigma E \)). The boundary condition for electric field on the PEC surface is that the tangential electric field should be zero.

In Fig.1, a rectangular waveguide is shown together with its conventional coordinate system and dimensions \((a \geq b)\).

Fig. 1. The rectangular waveguide.

Since we are seeking solutions of electromagnetic fields in the form of harmonic functions, the electric/magnetic fields inside the waveguide can be expressed in terms of sine, cosine, and complex exponential functions. The sine and cosine functions are representing standing waves while the complex exponential functions are representing propagation waves.
It is also convenient to distinguish TE and TM waves in a waveguide. In this lab, we are concerned with the TE mode only (For TE modes, the electric field is transverse to the waveguide axis, \( z \)-axis in Fig. 1). Thus, the electric field does not have \( z \)-component; or we can only have \( E_x \) and \( E_y \) components in the waveguide shown in Fig. 1.

The electric and magnetic field components of TE modes can be expressed

\[
E_{x,mn} = A_{mn} \cos \left( \frac{m \pi}{a} x \right) \sin \left( \frac{n \pi}{b} y \right) e^{-j \beta_z z}
\]

\[
E_{y,mn} = -B_{mn} \sin \left( \frac{m \pi}{a} x \right) \cos \left( \frac{n \pi}{b} y \right) e^{-j \beta_z z}
\]

\[
H_{x,mn} = -\frac{\beta_z E_y}{\omega \mu}
\]

\[
H_{y,mn} = \frac{\beta_z E_x}{\omega \mu}
\]

\[
H_{z,mn} = -j C_{mn} \cos \left( \frac{m \pi}{a} x \right) \cos \left( \frac{n \pi}{b} y \right) e^{-j \beta_z z}
\]

A pair of \((m, n)\)-values determines a mode in the waveguide; here \( m, n = 0, 1, 2 \ldots \) and \( m \) and \( n \) are not equal to zero simultaneously. A thorough understanding of the field configuration (spatial variations) of a mode is very important for the design of a waveguide. For us engineers, “a picture is worth a thousand words” is quite true and especially useful. So a proper visualization of the field distribution for each mode in a waveguide is a convenient and practically valuable means for us.

In electrical engineering, the electric field inside a waveguide is usually described by field lines with arrows: one set of lines for electric field and another set for magnetic field. Since the field distribution is three-dimensional, it is conventional to draw the field distribution on several cross sections in the waveguide. These cross sections are typically parallel to each of the three coordinate planes, i.e., the \( xy \), \( yz \), and \( zx \) planes.

In our textbook, Section (8.5) discusses the field configuration and how to represent it using field lines. In Appendix A, a copy of section 8.5 in the textbook is replicated.

2. Visualization of E- and H-fields in a rectangular waveguide: An alternative

We can also use a different way to visualize the electric and magnetic field in a waveguide. In the software of this lab, these fields are calculated on three cross sections
which can be moved along corresponding axes. The field on each of these cross sections is sampled on a square grid. For example, the grid on a cross-sectional plane parallel to the \(xy\)-plane, we have a grid as shown in Fig. 2.

![Image](image1.png)

Fig. 2. The square grid (left) on which the EM field is sampled (right). The red arrows are for E-field and the green ones are for H-field.

The E- and H-fields at each grid point are calculated and represented by arrows. These arrows can represent the field strength (a heavier arrow stands for stronger field) and the direction at every sampling point, as shown in Fig. 2 where E-field is represented by red arrows while H-field is represented by green arrows.

Fig. 3 is an illustration of the field distributions for \(TE_{10}\) mode in the waveguide at three planes: \(x = 0\), \(y = 0\), and \(z = 0\).

![Image](image2.png)

Fig. 3. \(TE_{10}\) mode in a rectangular waveguide.

The use of vectors to represent EM field at each sampling point can tell us directly the field (relative) strength and direction at these points. The obvious drawback is the missing of field lines which are the first tangible ‘things’ when we started learning
electromagnetics. In this lab we will obtain these field lines based on the visible vector representation of the EM fields.

3. Use of the Software

When the software is running, you need to input two parameters, $m$ and $n$ which are the mode indices. In Fig. 4, several screen shots are shown when the software starts ($m = 1, n = 0$ in this case). We first see (a) after the $m$ and $n$ values are set. Then with one mouse click, we get (b); one more click, we get (c); and another click gives us (d).

Fig. 3. (a) The waveguide displayed at the beginning of the software. (b) The waveguide when the walls are taken off and three cross-sectional planes are shown. The EM fields on these planes will be visualized. (c) The cross-sectional planes are taken away with their outlines kept. (d) The EM fields on these cross-sectional planes.

The length of the waveguide is always scaled to one waveguide wavelength for better visualization. Three outlines are plotted to divide the waveguide in $z$-direction into four equal parts (each of 1/4 waveguide wavelength), as shown in Fig. 3 (d).

A List of Functions of the Software:

General:
(i) Rotation: right click and drag;
(ii) Zoom in/out: press both mouse buttons and drag;
(iii) Animation on/off: key ‘t’;

Change the sizes of the field/current arrows. In some cases, the arrows are two small/large. The following functions help us adjust the arrow sizes for better visualization:

(iv) Thickening arrows: key ‘+’; (plus)
(v) Thinning arrows: key ‘-’; (minus)
(vi) Thickening E-field arrows only: key ‘E’;

We can view/hide the 4 plates of the waveguide using the following keys:

(vii) Upper plate on/off: key ‘u’;
(viii) Bottom plate on/off: key ‘b’;
(ix) Left plate on/off: key ‘l’;
(x) Right plate on/off: key ‘r’;

Some time we need to move the observation cross-sections to the center of the waveguide:

(xi) Set XP to x=a/2: key ‘x’;
(xii) Set YP to y=b/2: key ‘y’;
(xiii) Set ZP to z=λ/2: key ‘z’;

or hide/show these cross-sections:

(xiv) Toggle XP: key ‘X’;  
(xv) Toggle YP: key ‘Y’; 
(xvi) Toggle ZP: key ‘Z’;  

Good for drawing field lines

We can also hide/show the electric and magnetic field:

(xvii) Toggle e-field: key ‘e’
(xviii) Toggle h-field: key ‘h’

To reset the time and change the mode:

(xix) Reset time = 0: key ‘5’;
(xx) Change mode: key ‘m’