Distributed Optimal Resource Allocation for Fading Relay Broadcast Channels

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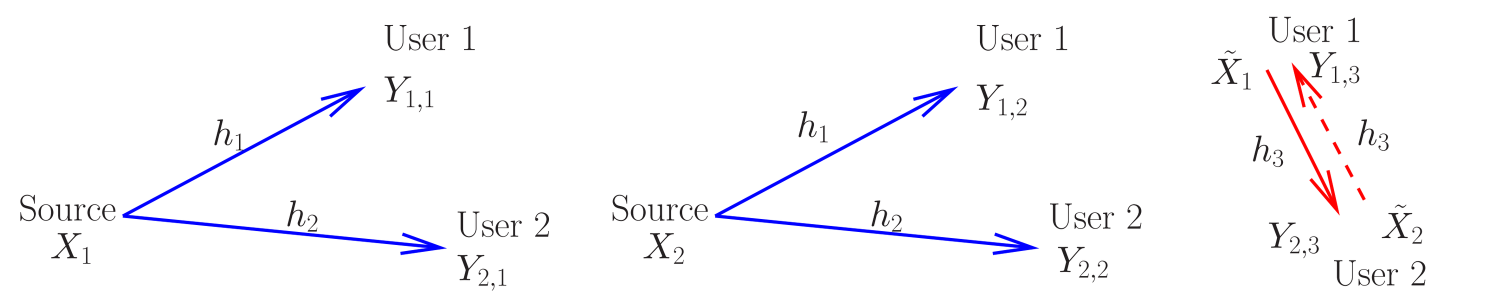
1 Abstract

Resource allocation for a fading orthogonal relay broadcast channel model is investigated, where the source broadcasts to two users using a time-division (TD) scheme in one channel, and the two users transmit relay signals to each other in another orthogonal channel. The channel bandwidth resource is assumed to be allocated equally for broadcast and relay transmissions. Separate power constraints are assumed at the source and the two users. The fading state information is assumed to be known at both the transmitter and receiver of each link, and hence the source and the two users can allocate their power adaptively according to the instantaneous channel state information in a distributed manner. For a fixed allocation of time (for TD broadcast) and power, an achievable rate region based on the relays using the decode-and-forward scheme for this model is derived. The resource (time and power) allocation is then optimized to maximize each boundary point of this rate region.

2 System Model: Three Orthogonal Channels

- Channel I: source transmits only message for user 1, and both users listen
- Channel II: source transmits only message for user 2, and both users listen
- Channel III: user with better channel transmits relay signals to user with worse channel



Channel I **Channel II** **Channel III**

3 Orthogonal Relay TD Broadcast Channel

- **Relationships between Input-Output Symbols:**

$$Y_{1,1} = \sqrt{\rho_1}\, h_1 X_1 + Z_{1,1}, \qquad Y_{2,1} = \sqrt{\rho_2}\, h_2 X_1 + Z_{2,1},$$
$$Y_{1,2} = \sqrt{\rho_1}\, h_1 X_2 + Z_{1,2}, \qquad Y_{2,2} = \sqrt{\rho_2}\, h_2 X_2 + Z_{2,2},$$
$$Y_{1,3} = \sqrt{\rho_3}\, h_3 \tilde{X}_2 + Z_{1,3} \qquad Y_{2,3} = \sqrt{\rho_3}\, h_3 \tilde{X}_1 + Z_{2,3},$$

  - h_1, h_2, h_3: independent fading variables (not necessarily Gaussian) with variances equal to 1
  - $Z_{i,j}$: independent Gaussian random variables with variances equal to 1
  - ρ_1, ρ_2 and ρ_3: link gain to noise ratios
  - $\{X_{1,n}, X_{2,n}\}$, $\{\tilde{X}_{1,n}\}$, $\{\tilde{X}_{2,n}\}$: subject to separate average power constraints P, $\tilde{P}_1$ and $\tilde{P}_2$
- **Assumptions on Channel Model:**
  - Equal bandwidth allocation between broadcasting (channel I & II) and relaying (channel III)
  - Time allocations for broadcasting to user 1 (channel I) and user 2 (channel II) are τ_1 and τ_2 with $\tau_1 + \tau_2 = 1$.
  - Source and users know fading states of links on which they transmit
- **Goal:**
  - Finding jointly optimal power allocations at source and relays and time allocations τ_1 and τ_2 that maximize achievable rate region

4 Relaying Scheme

- Relays (two users) use decode-and-forward scheme
- Define set $A := \{\underline{h} : \rho_1|h_1|^2 > \rho_2|h_2|^2\}$
- If $\underline{h} \in A$, user 1 assists user 2 by sending relay signals in channel III
- If $\underline{h} \in A^c$, user 2 assists user 1 by sending relay signals in channel III

5 Achievable Rate Region

Theorem 1. *An achievable rate region for the orthogonal relay broadcast channel model based on the TD broadcast scheme is given by*

$$\mathcal{R}(P, \tilde{P}_1, \tilde{P}_2) = \bigcup_{\mathcal{P} \in \mathcal{G}} \{(R_1, R_2) \text{ that satisfy following constraints on } R_1 \text{ and } R_2\}.$$

Furthermore, region $\mathcal{R}(P, \tilde{P}_1, \tilde{P}_2)$ is a convex set.

$$R_1 \le \frac{1}{2}\min\Big\{ \mathrm{E}_A\big[\tau_1(\underline{h})\mathcal{C}(\rho_1|h_1|^2P_1(\underline{h}))\big] + \mathrm{E}_{A^c}\big[\mathcal{C}(\rho_3|h_3|^2\tilde{P}_2(\underline{h}))\big] + \mathrm{E}_{A^c}\big[\tau_1(\underline{h})\mathcal{C}(\rho_1|h_1|^2P_1(\underline{h}))\big],$$
$$\mathrm{E}_A\big[\tau_1(\underline{h})\mathcal{C}(\rho_1|h_1|^2P_1(\underline{h}))\big] + \mathrm{E}_{A^c}\big[\tau_1(\underline{h})\mathcal{C}(\rho_2|h_2|^2P_1(\underline{h}))\big]\Big\}$$

$$R_2 \le \frac{1}{2}\min\Big\{ \mathrm{E}_A\big[\tau_2(\underline{h})\mathcal{C}(\rho_2|h_2|^2P_2(\underline{h}))\big] + \mathrm{E}_A\big[\mathcal{C}(\rho_3|h_3|^2\tilde{P}_1(\underline{h}))\big] + \mathrm{E}_{A^c}\big[\tau_2(\underline{h})\mathcal{C}(\rho_2|h_2|^2P_2(\underline{h}))\big],$$
$$\mathrm{E}_A\big[\tau_2(\underline{h})\mathcal{C}(\rho_1|h_1|^2P_2(\underline{h}))\big] + \mathrm{E}_{A^c}\big[\tau_2(\underline{h})\mathcal{C}(\rho_2|h_2|^2P_2(\underline{h}))\big]\Big\}$$

where power and time allocation functions satisfy:

$$\frac{1}{2}\mathrm{E}[\tau_1(\underline{h})P_1(\underline{h}) + \tau_2(\underline{h})P_2(\underline{h})] \le P, \qquad \frac{1}{2}\mathrm{E}_A[\tilde{P}_1(\underline{h})] \le \tilde{P}_1, \qquad \frac{1}{2}\mathrm{E}_{A^c}[\tilde{P}_2(\underline{h})] \le \tilde{P}_2, \tag{1}$$
$$\tau_1(\underline{h}) + \tau_2(\underline{h}) = 1, \qquad \text{for any } \underline{h}.$$

and $\mathcal{P}$ and $\mathcal{G}$ are defined as:

$$\mathcal{P} := \{P_1(\underline{h}), P_2(\underline{h}), \tilde{P}_1(\underline{h}), \tilde{P}_2(\underline{h}), \tau_1(\underline{h}), \tau_2(\underline{h})\},$$
$$\mathcal{G} := \{\text{all } \mathcal{P} \text{ that satisfy resource constraints given in (1)}\}.$$

6 Characterization of Achievable Rate Region

- **Boundary of $\mathcal{R}(P, \tilde{P}_1, \tilde{P}_2)$ is Characterized as:**

Since $\mathcal{R}(P, \tilde{P}_1, \tilde{P}_2)$ is convex, for any point (R_1, R_2) on the boundary of $\mathcal{R}(P, \tilde{P}_1, \tilde{P}_2)$, there exist $\mu_1, \mu_2 > 0$, such that (R_1, R_2) is a maximization solution to

$$\max_{(R_1,R_2)\in\mathcal{R}(P,\tilde{P}_1,\tilde{P}_2)} \mu_1 R_1 + \mu_2 R_2.$$

- **Main Goal:**

Finding optimal power and time allocation rules that achieve each point on the boundary of $\mathcal{R}(P, \tilde{P}_1, \tilde{P}_2)$, i.e., for any given $\mu_1, \mu_2 > 0$

$$\max_{\mathcal{P}\in\mathcal{G}} L(\mu_1, \mu_2) := \mu_1 \min\{R_{11}(\mathcal{P}), R_{12}(\mathcal{P})\} + \mu_2 \min\{R_{21}(\mathcal{P}), R_{22}(\mathcal{P})\}$$

7 Optimal Power and Time Allocation Rules

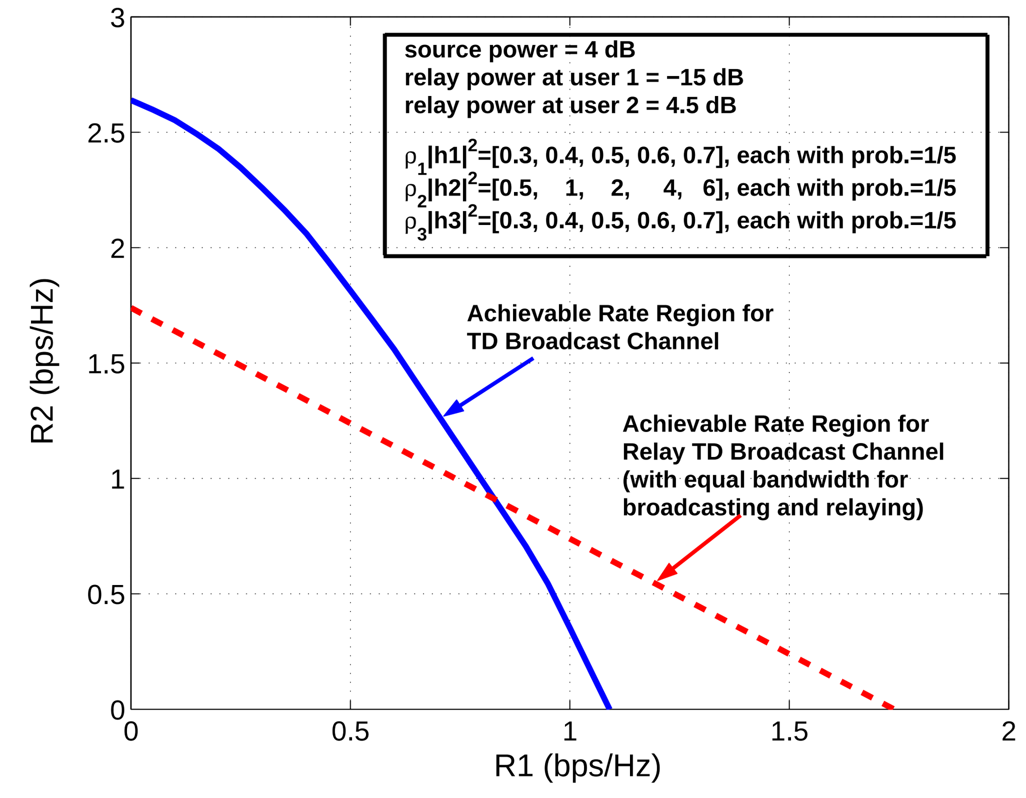
Three Cases for Optimal $\mathcal{P}^*$:

- **Case A, $\tilde{P}_1 < \tilde{P}_{1,l}$:**
  - $\mathcal{P}^*$ optimizes $L(\mu_1, \mu_2) = \mu_1 R_{12}(\mathcal{P}) + \mu_2 R_{21}(\mathcal{P})$
  - Threshold $\tilde{P}_{1,l}$ is determined by $R_{21}(\mathcal{P}^*) < R_{22}(\mathcal{P}^*)$
- **Case B, $\tilde{P}_1 > \tilde{P}_{1,u}$:**
  - $\mathcal{P}^*$ optimizes $L(\mu_1, \mu_2) = \mu_1 R_{12}(\mathcal{P}) + \mu_2 R_{22}(\mathcal{P})$
  - Threshold $\tilde{P}_{1,u}$ is determined by $R_{21}(\mathcal{P}^*) > R_{22}(\mathcal{P}^*)$
- **Case C, $\tilde{P}_{1,l} \le \tilde{P}_1 \le \tilde{P}_{1,u}$:**
  - $\mathcal{P}^*$ optimizes $L(\mu_1, \mu_2) = \mu_1 R_{11}(\mathcal{P}) + \mu_2 R_{21}(\mathcal{P})$
  - $\mathcal{P}^*$ satisfies $R_{11}(\mathcal{P}^*) = R_{12}(\mathcal{P}^*)$, $R_{21}(\mathcal{P}^*) = R_{22}(\mathcal{P}^*)$

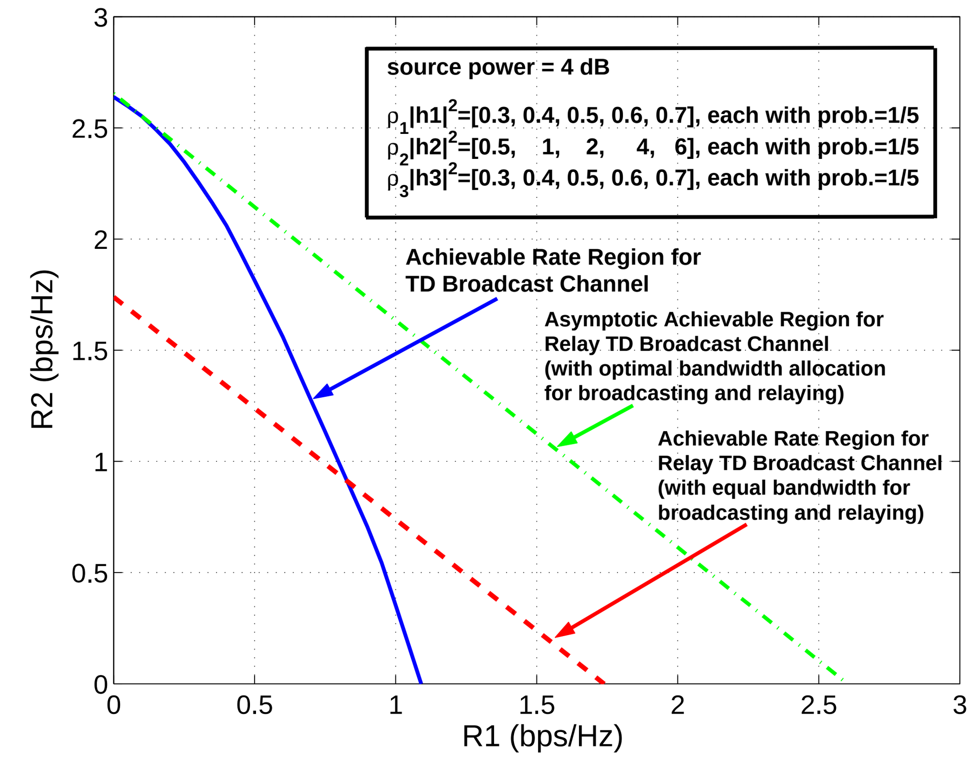
Properties of Optimal $\mathcal{P}^*$:

- Optimal power allocations at relays have water-filling forms according to h_3 for all three cases
- Optimal power and time allocations at source have more complicated forms and depend on both h_1 and h_2, refer to proceedings for details

8 Comparison of Achievable Rate Regions



9 Rate Region by Using Optimal Bandwidth Allocation for Broadcasting (Channel I & II) and Relaying (Channel III)



10 Conclusions

- Weaker user gets more improvement in rate due to relaying
- Equal bandwidth allocation for broadcasting and relaying enlarges only part of rate region corresponding to weaker user
- Optimizing bandwidth allocation between broadcasting and relaying enlarges entire rate region, but has more complexity

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