

Distributed Optimal Resource Allocation for Fading Relay Broadcast Channels

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Abstract—Resource allocation for a fading orthogonal relay broadcast channel model is investigated, where the source broadcasts to two users using a time-division (TD) scheme in one channel, and the two users transmit relay signals to each other in another orthogonal channel. The channel bandwidth resource is assumed to be allocated equally for broadcast and relay transmissions. Separate power constraints are assumed at the source and the two users. The fading state information is assumed to be known at both the transmitter and receiver of each link, and hence the source and the two users can allocate their power adaptively according to the instantaneous channel state information in a distributed manner. For a fixed allocation of time (for TD broadcast) and power, an achievable rate region based on the relays using the decode-forward scheme for this model is derived. The resource (time and power) allocation is then optimized to maximize each boundary point of this rate region.

I. INTRODUCTION

In centralized wireless networks such as cellular and WiFi data networks, mobile users have been demanding increasingly higher data rates on the downlink, particularly for Web applications. This motivates the study of downlink systems that incorporate relay links to achieve higher throughputs. Two basic relay broadcast systems have been recently introduced and studied. In the first system, relay links are introduced between the users in the broadcast system [1]–[3]. In the second system, an additional relay node is introduced into the broadcast channel that assists all the users [2]–[4]. The achievable regions for these channels are derived and the capacity improvements offered by user cooperation and relaying are investigated.

In this paper, we study a two-user fading relay broadcast system, where the source has independent messages for the two users in the system, and the users can transmit to each other through relay links. We impose the practical constraint that the relays (users) should transmit and receive in orthogonal channels. In this orthogonal model, the source transmits to users 1 and 2 (broadcast transmission) in one channel, and the two users take turns to transmit to each other (relay transmission) in another orthogonal channel. We assume that the channel bandwidth resources are allocated equally for broadcast and relay transmissions. The analysis can be

generalized to include the optimization of the allocation of the bandwidth resources between the two type of transmissions as in [5].

We make the restriction that the source uses a time-division (TD) scheme for the broadcast transmission to the two users. We further assume that each transmission link is corrupted by a multiplicative stationary and ergodic fading process as well as an additive white Gaussian noise process. Each node (source and two users) is assumed to know the fading state information of the links over which it transmits or receives, so that it can allocate the transmission power adaptively according to the instantaneous fading state information. We assume that the source and two users are subject to separate power constraints instead of a total power constraint. This assumption is more practical because the source and two users are usually geographically separated and hence are supported by separate power supplies. Furthermore, this assumption allow the nodes in the system to allocate their powers in a distributed manner, which is preferable for practical implementation. Our goal in this paper is to jointly optimize the allocation of the power at the nodes and the time division for broadcast transmission using the achievable rate region as the metric for optimization.

The resource allocation for the fading broadcast channel has been previously studied in [6], [7]. In particular, the TD broadcast channel has been investigated in [7]. We generalize this analysis to orthogonal relay broadcast channels, and investigate the impact of user cooperation on the performance of the broadcast channel. We first derive an achievable rate region for the orthogonal relay broadcast channel. We further show that the rate region is convex and derive an optimal resource allocation that maximizes each point on the boundary of this rate region.

In the following sections, we first introduce the system model, and then present our main results on resource allocation for the fading orthogonal relay broadcast channel.

II. SYSTEM MODEL

The system model for the orthogonal relay broadcast channel is illustrated in Figure 1. In channel I, the source transmits only the message for user 1, and both users 1 and 2 can listen. In channel II, the source transmits only the message for user 2, and both users can listen. The time allocation for these channels are represented as τ_1 and τ_2 with $\tau_1 + \tau_2 = 1$. In channel III, the users take turns to transmit relay signals, with

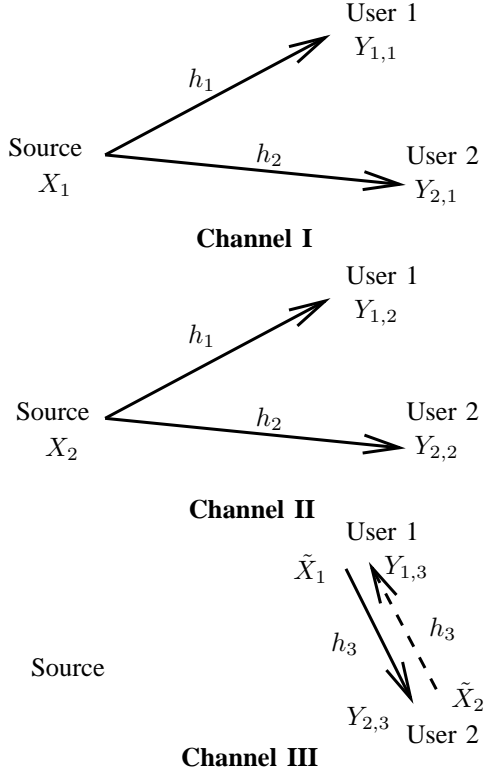


Fig. 1. Fading Orthogonal Relay TD Broadcast Model

the user with the better channel from the source helping the other user.

We use X_1 and X_2 to denote the signals sent from the source in channels I and II, respectively, and use \tilde{X}_1 and \tilde{X}_2 to denote the signals sent from the users 1 and 2, respectively. We use $Y_{1,1}, Y_{1,2}, Y_{1,3}$ to denote the signals received at the user 1 in the three channels, respectively, and use $Y_{2,1}, Y_{2,2}, Y_{2,3}$ to denote the signals received at the user 2 in the three channels, respectively. We use h_1 and h_2 to denote the fading coefficients corresponding to transmission links from the source to user 1 and from the source to user 2, respectively. We further assume that the transmission links between user 1 and user 2 are same, and use h_3 to denote the common fading coefficient. For notational convenience, we collect the three fading coefficients in a vector $\underline{h} := (h_1, h_2, h_3)$.

The relationships between the input and output symbols in the three orthogonal channels at each symbol time instant can be written as

$$\begin{aligned}
 Y_{1,1} &= \sqrt{\rho_1} h_1 X_1 + Z_{1,1}, \\
 Y_{2,1} &= \sqrt{\rho_2} h_2 X_1 + Z_{2,1}, \\
 Y_{1,2} &= \sqrt{\rho_1} h_1 X_2 + Z_{1,2}, \\
 Y_{2,2} &= \sqrt{\rho_2} h_2 X_2 + Z_{2,2}, \\
 Y_{1,3} &= \sqrt{\rho_3} h_3 \tilde{X}_2 + Z_{1,3}, \\
 Y_{2,3} &= \sqrt{\rho_3} h_3 \tilde{X}_1 + Z_{2,3},
 \end{aligned} \tag{1}$$

where h_1, h_2, h_3 are independent zero mean complex proper random variables (not necessarily Gaussian) with variances

normalized to 1, and $Z_{i,j}$ are independent zero mean Gaussian random variables with variances also normalized to 1. Then the parameters ρ_1, ρ_2 and ρ_3 represent the link gain to noise ratios of the corresponding transmission links. The input symbol sequences $\{X_{1,n}, X_{2,n}\}$, $\{\tilde{X}_{1,n}\}$ and $\{\tilde{X}_{2,n}\}$ are subject to separate average power constraints P, \tilde{P}_1 and \tilde{P}_2 , respectively.

Since the source and users (relays) know the fading states of the links on which they transmit, they can allocate their transmitted signal powers according to the channel state information to achieve the best performance. As we pointed out earlier, the time allocation parameters τ_1 and τ_2 between the two users also need to be optimized jointly with the power allocation.

III. MAIN RESULTS

In this section, we first derive an achievable rate region for the orthogonal relay TD broadcast channel, and then optimize the resource allocation to maximize each point on the boundary of the achievable rate region.

For notational convenience, we first define a set $A := \{\underline{h} : \rho_1|h_1|^2 > \rho_2|h_2|^2\}$, which contains all the fading states \underline{h} with the link from the source to user 1 being better than the link from the source to user 2. The complement of the set A is $A^c := \{\underline{h} : \rho_1|h_1|^2 \leq \rho_2|h_2|^2\}$. We derive an achievable rate region based on the following scheme. If the link from the source to user 1 is better than the link from the source to user 2, i.e., $\underline{h} \in A$, user 1 assists user 2 by sending a relay signal in channel III. If $\underline{h} \in A^c$, then user 2 assists user 1 by sending a relay signal in channel III. Hence at each time instant, only one user transmits relay signals to the other. Both user 1 and user 2 use the decode-forward scheme [8, Sec. II] when sending relay signals. Based on this transmission scheme, for a fixed \underline{h} , we have the following achievable rate region.

If $\underline{h} \in A$,

$$\begin{aligned}
 R_1(\underline{h}) &\leq \frac{1}{2} \tau_1(\underline{h}) \mathcal{C}(\rho_1|h_1|^2 P_1(\underline{h})) \\
 R_2(\underline{h}) &\leq \min \left\{ \frac{1}{2} \tau_2(\underline{h}) \mathcal{C}(\rho_2|h_2|^2 P_2(\underline{h})) + \frac{1}{2} \mathcal{C}(\rho_3|h_3|^2 \tilde{P}_1(\underline{h})), \right. \\
 &\quad \left. \frac{1}{2} \tau_2(\underline{h}) \mathcal{C}(\rho_1|h_1|^2 P_2(\underline{h})) \right\}
 \end{aligned}$$

where the function $\mathcal{C}(x) := \log(1+x)$, and $\tau_1(\underline{h}) + \tau_2(\underline{h}) = 1$. For a given fading state \underline{h} , the functions $P_1(\underline{h})$ and $P_2(\underline{h})$ are the powers used by the source to transmit information for users 1 and 2, respectively, and $\tilde{P}_1(\underline{h})$ is the power used by user 1 to transmit relay signals for user 2.

If $\underline{h} \in A^c$,

$$\begin{aligned}
 R_1(\underline{h}) &\leq \min \left\{ \frac{1}{2} \tau_1(\underline{h}) \mathcal{C}(\rho_1|h_1|^2 P_1(\underline{h})) + \frac{1}{2} \mathcal{C}(\rho_3|h_3|^2 \tilde{P}_2(\underline{h})), \right. \\
 &\quad \left. \frac{1}{2} \tau_1(\underline{h}) \mathcal{C}(\rho_2|h_2|^2 P_1(\underline{h})) \right\}
 \end{aligned}$$

$$R_2(\underline{h}) \leq \frac{1}{2} \tau_2(\underline{h}) \mathcal{C}(\rho_2|h_2|^2 P_2(\underline{h}))$$

where $\tau_1(\underline{h}) + \tau_2(\underline{h}) = 1$. For a given fading state \underline{h} , the function $\tilde{P}_2(\underline{h})$ is the power used by user 2 to transmit relay signals for user 1.

For fading channels with \underline{h} being a stationary and ergodic vector process, an achievable rate region is given by taking averages over all fading states, and can be written in the following form.

$$\begin{aligned}
R_1(\underline{h}) &\leq \frac{1}{2} \min \\
&\left\{ \begin{aligned} &E_A [\tau_1(\underline{h})\mathcal{C}(\rho_1|h_1|^2P_1(\underline{h}))] + E_{A^c} [\mathcal{C}(\rho_3|h_3|^2\tilde{P}_2(\underline{h}))] \\ &+ E_{A^c} [\tau_1(\underline{h})\mathcal{C}(\rho_1|h_1|^2P_1(\underline{h}))], \\ &E_A [\tau_1(\underline{h})\mathcal{C}(\rho_1|h_1|^2P_1(\underline{h}))] + E_{A^c} [\tau_1(\underline{h})\mathcal{C}(\rho_2|h_2|^2P_1(\underline{h}))] \end{aligned} \right\} \quad (2) \\
R_2(\underline{h}) &\leq \frac{1}{2} \min \\
&\left\{ \begin{aligned} &E_A [\tau_2(\underline{h})\mathcal{C}(\rho_2|h_2|^2P_2(\underline{h}))] + E_A [\mathcal{C}(\rho_3|h_3|^2\tilde{P}_1(\underline{h}))] \\ &+ E_{A^c} [\tau_2(\underline{h})\mathcal{C}(\rho_2|h_2|^2P_2(\underline{h}))], \\ &E_A [\tau_2(\underline{h})\mathcal{C}(\rho_1|h_1|^2P_2(\underline{h}))] + E_{A^c} [\tau_2(\underline{h})\mathcal{C}(\rho_2|h_2|^2P_2(\underline{h}))] \end{aligned} \right\} \quad (3)
\end{aligned}$$

where the power and time allocation functions satisfy

$$\begin{aligned}
\frac{1}{2}E[\tau_1(\underline{h})P_1(\underline{h}) + \tau_2(\underline{h})P_2(\underline{h})] &\leq P \\
\frac{1}{2}E_A[\tilde{P}_1(\underline{h})] &\leq \tilde{P}_1 \\
\frac{1}{2}E_{A^c}[\tilde{P}_2(\underline{h})] &\leq \tilde{P}_2 \\
\tau_1(\underline{h}) + \tau_2(\underline{h}) &= 1, \quad \text{for any } \underline{h}.
\end{aligned} \quad (4)$$

We now define

$$\mathcal{P} := \{P_1(\underline{h}), P_2(\underline{h}), \tilde{P}_1(\underline{h}), \tilde{P}_2(\underline{h}), \tau_1(\underline{h}), \tau_2(\underline{h})\},$$

$$\mathcal{G} := \{\text{all } \mathcal{P} \text{ that satisfy resource constraints given in (4)}\}.$$

Then we have the following theorem for the achievable rate region.

Theorem 1: An achievable rate region for the orthogonal relay broadcast channel model based on the TD broadcast scheme is given by

$$\mathcal{R}(P, \tilde{P}_1, \tilde{P}_2) = \bigcup_{\mathcal{P} \in \mathcal{G}} \{(R_1, R_2) \text{ that satisfies (2) and (3)}\}.$$

Further more, the region $\mathcal{R}(P, \tilde{P}_1, \tilde{P}_2)$ is a convex set.

Since the achievable region $\mathcal{R}(P, \tilde{P}_1, \tilde{P}_2)$ is convex, the boundary of $\mathcal{R}(P, \tilde{P}_1, \tilde{P}_2)$ can be characterized as follows. For any point (R_1, R_2) on the boundary of $\mathcal{R}(P, \tilde{P}_1, \tilde{P}_2)$, there exist $\mu_1, \mu_2 > 0$, such that (R_1, R_2) is a maximization solution to

$$\max_{(R_1, R_2) \in \mathcal{R}(P, \tilde{P}_1, \tilde{P}_2)} \mu_1 R_1 + \mu_2 R_2. \quad (5)$$

Our main goal in this paper is to find optimal power and time allocation rules that achieve each point on the boundary of $\mathcal{R}(P, \tilde{P}_1, \tilde{P}_2)$, i.e.,

$$\max_{\mathcal{P} \in \mathcal{G}} L(\mu_1, \mu_2) := \mu_1 R_1 + \mu_2 R_2 \text{ for any given } \mu_1, \mu_2 > 0$$

This optimization will serve as a complete characterization of the boundary of the region $\mathcal{R}(P, \tilde{P}_1, \tilde{P}_2)$. Throughout the following analysis, we assume $\mu_1 < \mu_2$. The case of $\mu_2 < \mu_1$ is handled similarly by switching the roles of user 1 and user 2.

Now let $R_{11}(\mathcal{P})$ and $R_{12}(\mathcal{P})$ respectively denote the two terms over which the minimization in (2) is taken, and let $R_{21}(\mathcal{P})$ and $R_{22}(\mathcal{P})$ respectively denote the two terms over which the minimization in (3) is taken. Then

$$\begin{aligned}
L(\mu_1, \mu_2) &= \mu_1 \min\{R_{11}(\mathcal{P}), R_{12}(\mathcal{P})\} + \mu_2 \min\{R_{21}(\mathcal{P}), R_{22}(\mathcal{P})\}
\end{aligned}$$

The optimal solution \mathcal{P}^* that maximizes $L(\mu_1, \mu_2)$ must fall into one of the following nine cases depending the power constraints at the source and users 1 and 2.

- Case 1: \mathcal{P}^* maximizes $\mu_1 R_{11} + \mu_2 R_{21}$;
with $R_{11}(\mathcal{P}^*) < R_{12}(\mathcal{P}^*)$, $R_{21}(\mathcal{P}^*) < R_{22}(\mathcal{P}^*)$
- Case 2: \mathcal{P}^* maximizes $\mu_1 R_{12} + \mu_2 R_{21}$;
with $R_{11}(\mathcal{P}^*) > R_{12}(\mathcal{P}^*)$, $R_{21}(\mathcal{P}^*) < R_{22}(\mathcal{P}^*)$
- Case 3: \mathcal{P}^* maximizes $\mu_1 R_{11} + \mu_2 R_{21}$;
with $R_{11}(\mathcal{P}^*) = R_{12}(\mathcal{P}^*)$, $R_{21}(\mathcal{P}^*) < R_{22}(\mathcal{P}^*)$
- Case 4: \mathcal{P}^* maximizes $\mu_1 R_{11} + \mu_2 R_{22}$;
with $R_{11}(\mathcal{P}^*) < R_{12}(\mathcal{P}^*)$, $R_{21}(\mathcal{P}^*) > R_{22}(\mathcal{P}^*)$
- Case 5: \mathcal{P}^* maximizes $\mu_1 R_{12} + \mu_2 R_{22}$;
with $R_{11}(\mathcal{P}^*) > R_{12}(\mathcal{P}^*)$, $R_{21}(\mathcal{P}^*) > R_{22}(\mathcal{P}^*)$
- Case 6: \mathcal{P}^* maximizes $\mu_1 R_{11} + \mu_2 R_{22}$;
with $R_{11}(\mathcal{P}^*) = R_{12}(\mathcal{P}^*)$, $R_{21}(\mathcal{P}^*) > R_{22}(\mathcal{P}^*)$
- Case 7: \mathcal{P}^* maximizes $\mu_1 R_{11} + \mu_2 R_{21}$;
with $R_{11}(\mathcal{P}^*) < R_{12}(\mathcal{P}^*)$, $R_{21}(\mathcal{P}^*) = R_{22}(\mathcal{P}^*)$
- Case 8: \mathcal{P}^* maximizes $\mu_1 R_{12} + \mu_2 R_{21}$;
with $R_{11}(\mathcal{P}^*) > R_{12}(\mathcal{P}^*)$, $R_{21}(\mathcal{P}^*) = R_{22}(\mathcal{P}^*)$
- Case 9: \mathcal{P}^* maximizes $\mu_1 R_{11} + \mu_2 R_{21}$;
with $R_{11}(\mathcal{P}^*) = R_{12}(\mathcal{P}^*)$, $R_{21}(\mathcal{P}^*) = R_{22}(\mathcal{P}^*)$

For each of the above cases, we can derive the optimal \mathcal{P}^* . However, only those cases with the corresponding conditions at \mathcal{P}^* being satisfied can possibly happen. We next give two lemmas that we will apply later for the optimization in each case.

Lemma 1: (Extended Version of [7, Theorem 3])

Consider

$$\begin{aligned}
\mathcal{J}(\mathcal{P}) = E &\left[\tau_1(\underline{h}) \left[a_1 \mathcal{C}(b_1 P_1(\underline{h})) + a_2 \mathcal{C}(b_2 P_1(\underline{h})) \right] \right. \\
&\left. + \tau_2(\underline{h}) a_3 \mathcal{C}(b_3 P_2(\underline{h})) \right] \quad (6)
\end{aligned}$$

where $\mathcal{P} = (P_1(\underline{h}), P_2(\underline{h}), \tau_1(\underline{h}), \tau_2(\underline{h}))$ needs to satisfy the constraints given in (4). The parameters a_i and b_i are positive real numbers. The optimal \mathcal{P}^* that maximizes $\mathcal{J}(\mathcal{P})$ is given as follows.

Let

$$\begin{aligned} f_1(p) &= a_1\mathcal{C}(b_1p) + a_2\mathcal{C}(b_2p) \\ f_2(p) &= a_3\mathcal{C}(b_3p) \end{aligned}$$

Then $f_1(p) = f_2(p)$ has at most two roots. According to the number of the roots that $f_1(p) = f_2(p)$ has, the optimization of $\mathcal{J}(\mathcal{P})$ has six possibilities.

(a) No roots and $f_1(p) < f_2(p)$, then

$$\begin{aligned} \tau_1^*(\underline{h}) &= 0, & \tau_2^*(\underline{h}) &= 1; \\ P_1^*(\underline{h}) &= 0, & P_2^*(\underline{h}) &= \left(\frac{a_3}{\lambda} - \frac{1}{b_3}\right)^+, \end{aligned}$$

where λ is chosen to satisfy the constraints given in (4).

(b) No roots and $f_1(p) \geq f_2(p)$, then

$$\begin{aligned} \tau_1^*(\underline{h}) &= 1, & \tau_2^*(\underline{h}) &= 0; \\ P_1^*(\underline{h}) &= \begin{cases} \text{positive root of } \frac{a_1b_1}{1+b_1p} + \frac{a_2b_2}{1+b_2p} = \lambda \\ \text{if it exists} \\ 0, & \text{otherwise;} \end{cases} \\ P_2^*(\underline{h}) &= 0 \end{aligned}$$

where λ is chosen to satisfy the constraints given in (4).

(c) One root, and $f_1(p) < f_2(p)$ for $0 < p < p_0$ where p_0 is a certain positive number.

Let r_1 be the slope of the common tangent line of $f_1(p)$ and $f_2(p)$, and let p_a and p_b be the tangent points that satisfy

$$f_1'(p_a) = f_2'(p_b) = r_1.$$

The values of p_a, p_b and r_1 can be determined by

$$\begin{aligned} f_1'(p_a) &= \frac{a_1b_1}{1+b_1p_a} + \frac{a_2b_2}{1+b_2p_a} = r_1 \\ f_2'(p_b) &= \frac{a_3b_3}{1+b_3p_b} = r_1 \\ r_1 &= \frac{f_2(p_b) - f_1(p_a)}{p_b - p_a} \end{aligned} \quad (7)$$

Then the optimal \mathcal{P}^* are given as follows.

(i) If $\lambda > r_1$

$$\begin{aligned} \tau_1^*(\underline{h}) &= 0, & \tau_2^*(\underline{h}) &= 1; \\ P_1^*(\underline{h}) &= 0, & P_2^*(\underline{h}) &= \left(\frac{a_3}{\lambda} - \frac{1}{b_3}\right)^+ \end{aligned}$$

(ii) If $0 < \lambda < r_1$

$$\begin{aligned} \tau_1^*(\underline{h}) &= 1, & \tau_2^*(\underline{h}) &= 0; \\ P_1^*(\underline{h}) &= \begin{cases} \text{positive root of } \frac{a_1b_1}{1+b_1p} + \frac{a_2b_2}{1+b_2p} = \lambda, \\ \text{if it exists} \\ 0, & \text{otherwise;} \end{cases} \\ P_2^*(\underline{h}) &= 0 \end{aligned}$$

(iii) If $\lambda = r_1$

$$\begin{aligned} \tau_1^*(\underline{h}) &= \tau^*, & \tau_2^*(\underline{h}) &= 1 - \tau^*, & \tau^* &\in [0, 1] \\ P_1^*(\underline{h}) &= \begin{cases} \text{positive root of } \frac{a_1b_1}{1+b_1p} + \frac{a_2b_2}{1+b_2p} = \lambda, \\ \text{if it exists} \\ 0, & \text{otherwise;} \end{cases} \\ P_2^*(\underline{h}) &= \left(\frac{a_3}{\lambda} - \frac{1}{b_3}\right)^+ \end{aligned}$$

The parameter λ and τ^* are chosen to satisfy the constraints given in (4).

(d) One root, and $f_1(p) \geq f_2(p)$ for $0 < p < p_0$ where p_0 is a certain positive number.

The optimal \mathcal{P}^* is similarly given as in (c) with user 1 and user 2 switching their roles.

(e) Two roots, and $f_1(p) < f_2(p)$ for $0 < p < p_0$ where p_0 is a certain positive number.

There are two common tangent lines for $f_1(p)$ and $f_2(p)$. Let the slopes of these two common tangent lines be r_1 and r_2 with $r_1 > r_2$. These two slopes can be determined by solving (7).

Then the optimal \mathcal{P}^* are given as follows.

(i) If $\lambda > r_1$, \mathcal{P}^* is same as (c) (i);

(ii) If $r_2 < \lambda < r_1$, \mathcal{P}^* is same as (c) (ii);

(iii) If $\lambda < r_2$, \mathcal{P}^* is same as (c) (i);

(iv) If $\lambda = r_1$, \mathcal{P}^* is same as (c) (iii);

(v) If $\lambda = r_2$, \mathcal{P}^* is same as (c) (iii);

(f) Two roots, and $f_1(p) \geq f_2(p)$ for $0 < p < p_0$ where p_0 is a certain positive number.

The optimal \mathcal{P}^* is similarly given as in (e) with user 1 and user 2 switching roles.

For later reference, we write the water-filling solution for the simple optimization problem in the following lemma.

Lemma 2: (Water-filling Solution)

Consider

$$\begin{aligned} \mathcal{J}_1(\mathcal{P}) &= \mathbb{E}_A \mathcal{C}(\rho_3 |h_3|^2 \tilde{P}_1(\underline{h})) \\ \mathcal{J}_2(\mathcal{P}) &= \mathbb{E}_{A^c} \mathcal{C}(\rho_3 |h_3|^2 \tilde{P}_2(\underline{h})) \end{aligned} \quad (8)$$

where $\tilde{P}_1(\underline{h})$ and $\tilde{P}_2(\underline{h})$ need to satisfy the constraints given in (4). The optimal $\tilde{P}_1^*(\underline{h})$ and $\tilde{P}_2^*(\underline{h})$ that maximize $\mathcal{J}_1(\mathcal{P})$ and $\mathcal{J}_2(\mathcal{P})$ are given by

$$\begin{aligned} \tilde{P}_1^*(\underline{h}) &= \begin{cases} \left(\frac{1}{\lambda_1} - \frac{1}{\rho_3 |h_3|^2}\right)^+, & \text{if } \underline{h} \in A, \\ 0, & \text{if } \underline{h} \in A^c, \end{cases} \\ \tilde{P}_2^*(\underline{h}) &= \begin{cases} \left(\frac{1}{\lambda_2} - \frac{1}{\rho_3 |h_3|^2}\right)^+, & \text{if } \underline{h} \in A^c, \\ 0, & \text{if } \underline{h} \in A, \end{cases} \end{aligned} \quad (9)$$

where λ_1 and λ_2 are chosen to satisfy the power constraints given in (4).

Applying Lemmas 1 and 2, we can solve the optimization for the nine cases. It can be shown that only cases 2, 5, 9 can possibly happen. For the sake of brevity, we have omitted the details of the analysis. We summarize the solution \mathcal{P}^* that optimizes $L(\mu_1, \mu_2)$ in the following theorem.

Theorem 2: For $\mu_1 < \mu_2$, the optimal \mathcal{P}^* that maximizes $L(\mu_1, \mu_2)$ is given by the following three cases.

Case A. (Case 2) If $\tilde{P}_1 < \tilde{P}_{1,l}$, then \mathcal{P}^* is determined as follows.

For $\underline{h} \in A$, $\tau_1^*(\underline{h}), \tau_2^*(\underline{h}), P_1^*(\underline{h}), P_2^*(\underline{h})$ are given by Lemma 1 (a) and (c) with $a_1 = \mu_1, a_2 = 0, a_3 = \mu_2, b_1 = \rho_1|h_1|^2, b_2 = 0, b_3 = \rho_2|h_2|^2$;
 For $\underline{h} \in A^c$, $\tau_1^*(\underline{h}), \tau_2^*(\underline{h}), P_1^*(\underline{h}), P_2^*(\underline{h})$ are given by Lemma 1 (a) with $a_3 = \mu_2, b_3 = \rho_2|h_2|^2$;
 For any \underline{h} , $\tilde{P}_1^*(\underline{h})$ is given by Lemma 2.

The threshold $\tilde{P}_{1,l}$ is determined by the condition $R_{21}(\mathcal{P}^*) < R_{22}(\mathcal{P}^*)$.

Case B. (Case 5) If $\tilde{P}_1 > \tilde{P}_{1,u}$, then \mathcal{P}^* is determined as follows.

For $\underline{h} \in A$, $\tau_1^*(\underline{h}), \tau_2^*(\underline{h}), P_1^*(\underline{h}), P_2^*(\underline{h})$ are given by Lemma 1 (a) with $a_1 = \mu_1, a_2 = 0, a_3 = \mu_2, b_1 = \rho_1|h_1|^2, b_2 = 0, b_3 = \rho_1|h_1|^2$;
 For $\underline{h} \in A^c$, $\tau_1^*(\underline{h}), \tau_2^*(\underline{h}), P_1^*(\underline{h}), P_2^*(\underline{h})$ are given by Lemma 1 (a) with $a_1 = \mu_1, a_2 = 0, a_3 = \mu_2, b_1 = \rho_2|h_2|^2, b_2 = 0, b_3 = \rho_2|h_2|^2$;
 For any \underline{h} , $\tilde{P}_1^*(\underline{h})$ is given by Lemma 2.

The threshold $\tilde{P}_{1,u}$ is determined by the condition $R_{21}(\mathcal{P}^*) > R_{22}(\mathcal{P}^*)$.

Case C. (Case 9) If $\tilde{P}_{1,l} \leq \tilde{P}_1 \leq \tilde{P}_{1,u}$, then \mathcal{P}^* is determined as follows.

For $\underline{h} \in A$, $\tau_1^*(\underline{h}), \tau_2^*(\underline{h}), P_1^*(\underline{h}), P_2^*(\underline{h})$ are given by Lemma 1 (a) with $a_1 = \mu_1, a_2 = \mu_2(1 - \beta), a_3 = \mu_2\beta, b_1 = \rho_1|h_1|^2, b_2 = \rho_2|h_2|^2, b_3 = \rho_1|h_1|^2$;
 For $\underline{h} \in A^c$, $\tau_1^*(\underline{h}), \tau_2^*(\underline{h}), P_1^*(\underline{h}), P_2^*(\underline{h})$ are given by Lemma 1 (a) with $a_1 = \mu_1(1 - \alpha), a_2 = \mu_1\alpha, a_3 = \mu_2, b_1 = \rho_1|h_1|^2, b_2 = \rho_2|h_2|^2, b_3 = \rho_2|h_2|^2$;
 For any \underline{h} , $\tilde{P}_1^*(\underline{h}), \tilde{P}_2^*(\underline{h})$ are given by Lemma 2.

where α and β are chosen such that the conditions

$$R_{11}(\mathcal{P}^*) = R_{12}(\mathcal{P}^*), \quad R_{21}(\mathcal{P}^*) = R_{22}(\mathcal{P}^*)$$

are satisfied.

IV. CONCLUSIONS

We investigated the problem of the optimal resource (power and time) allocation for a fading orthogonal relay broadcast channel model. We showed that the optimal power allocation at the source and the users can be done in a distributed manner due to the assumption of separate power constraints at the different nodes in the system. We showed that the optimal power allocation for sending relay signals at the users is water-filling. For the source, the power allocation needs to be jointly

optimized with time allocation. We showed that in general the resource allocation at the source is more complicated than the resource allocation for the fading broadcast channel without relaying. However in some extreme cases, where the power constraints at the two users are very large or small compared to the source power constraints, the resource allocation at the source reduces to a form that is similar to that for the broadcast channel without relaying.

Numerical results that compare the performance of broadcast channels with and without relaying will be provided at the conference. Extensions to more general relay broadcast channels will also be discussed.

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