1. Basic Counting Principle.

Consider the following. Exp 1 has \( n_1 \) outcomes. For each outcome, Exp 2 offers \( n_2 \) outcomes.

Namely, whether the outcome in Exp 1 is 2 or \( n_1 \) or 1, Exp 2 has \( n_2 \) outcomes. Note that the actual outcomes in Exp 2 may depend on what happened in Exp 1—only the \# of outcomes in Exp 2 has to be the same no matter what happened in Exp 1. See Example Below.

Ex: Counting w/out replacement, \( n \) distinguishable objects, order important

If \( n = 3 \), \( k = 3 \)
Ex: Do the same for counting w/ replacement, n distinguishable objects, k trials, order important.

2. Corollary to the basic principle of counting

Suppose we have a "polygamous" map, n₁ objects on the left. Each object on the left is connected to d₁ objects on the right. Each object on the right is connected to d₂ objects on the left. Then the # of objects on the right, n₂, is given by

\[ n₂ = \frac{n₁ d₁}{d₂} \]

For example:

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1
2
3
4
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Proof:

We count the # of lines in 2 ways. Looking at the left side, # of lines = n₁d₁ (n₁ nodes, each has d₁ lines attached).

From the right, it is n₂d₂. But both count the same thing, so n₁d₁ = n₂d₂.

Example: Counting w/o replacement, order is not important, n distinguishable objects, k trials.

n = 3, k = 3
\[ n = 4, \quad k = 4 \]

The left degrees are \(4, 3, 2, 1\) right \(1, 2, 3, 4\).

In general, the left degrees are \(n, n-1, n-2, \ldots, 1\) right \(1, 2, 3, \ldots, n\).

For trial 1:

1. \(1 \cdot \frac{n-1}{1} = n\) ways.

2. \(n \cdot \frac{n-1}{2} = \binom{n}{2}\) ways.

3. \(\binom{n}{2} \cdot \frac{n-2}{3} = \binom{n}{3}\) ways.

4. \(\frac{n(n-1) \cdots (n-k+1)}{1 \cdot 2 \cdots k} \cdot \frac{k}{k} = \frac{n}{k} = \binom{n}{k}\) ways.