8.1 PreLab

1. Assuming that the step size of the uniform quantizer is $2^{-b}$, calculate the mean square value of the quantization noise.

2. What is the input-output characteristic of a $\mu$-law compressor and expander? Draw the characteristic of the compressor using MATLAB. In encoding of speech waveforms the value of $\mu = 255$ has been adopted as standard. How much reduction (in dB) does this value yield in the quantization noise power relative to uniform quantization?

3. What is an advantage of PCM over DPCM and vice-versa?

8.2 Overview

Pulse code modulation (PCM) is a method for quantizing an analog signal for the purpose of transmitting or storing the signal in digital form. PCM is widely used for speech transmission in telephone communications and for telemetry systems that employ radio transmission. We shall concentrate our attention on the application of PCM to speech signal processing.

Speech signals transmitted over telephone channels are usually limited in bandwidth to the frequency range below 4 kHz. Hence, the Nyquist rate for sampling such a signal is less than 8 kHz. In PCM, the analog speech signal is sampled at the nominal rate of 8 kHz, and each sample is quantized to one of $2^b$ levels and represented digitally by a sequence of $b$ bits.

The quantization process may be modeled mathematically as

$$s(n) = m(n) + q(n)$$  \hspace{1cm} (8.1)

where $s(n)$ represents the quantized value of $m(n)$, and $q(n)$ is the quantization error—which we treat as additive noise.
Speech signals have the characteristic that small signal amplitudes occur more frequently than large signal amplitudes. However, a uniform quantizer provides the same spacing between the successive levels throughout the entire dynamic range of the signal. A better approach is to use a non-uniform quantizer which provides more closely spaced levels at the low signal amplitudes and more widely spaced levels at the large signal amplitudes. A non-uniform quantizer characteristic is usually obtained by passing the signal through a nonlinear device to compress the signal amplitude and then a uniform quantizer.

### 8.3 Procedure

The purpose of this lab is to gain an understanding of PCM compression (linear to logarithmic) and PCM expansion (logarithmic to linear). Write the following MATLAB functions for this lab:

1. A $\mu$-law compressor function to implement the input-output magnitude characteristic that accepts a zero mean normalized ($|s| \leq 1$) signal and produces a compressed zero mean signal with $\mu$ as a free parameter that can be specified.

2. A quantizer function that accepts a zero mean input and produces an output (as shown in Figure 8.1) after $b$-bit quantization that can be specified.

3. A $\mu$-law expander to implement the input-output magnitude characteristic that accepts a signal and produces a zero mean output for a specified parameter $\mu$.

For simulation purposes, generate a large number of samples (at least 10,000) which are similar to speech signals. The following steps should give a simulated speech signal:

1. $x(n) \sim \mathcal{U}(0, 1)$
2. $w(n) \sim \mathcal{U}(-1, 1)$
3. $y(n) = w(n) \times \log(x(n))$
4. Pass \( y(n) \) through a low-pass filter with \( w_0 = 1 \) to get \( z(n) \)

5. Normalize \( z(n) \) to get \( m(n) \)

Process these samples through the \( \mu \)-law compressor, quantizer, and expander you created—as in Figure 8.1. Compute the signal-to-quantization noise ratio in dB:

\[
\text{SQNR} = 10 \log_{10} \left[ \frac{\sum_{n=1}^{N} m^2(n)}{\sum_{n=1}^{N} (m(n) - s(n))^2} \right]
\]

(8.2)

Compare this to the SQNR without companding. Also plot the input and output waveforms for both cases and comment on the results.

Finally, for \( b \in \{4, 6, 8\} \)-bit quantizers, experimentally (numerically) determine the value of \( \mu \) that maximizes the SQNR.