Signals & Systems
HW

Problem Set 1 (Due Wednesday 1/27)

Problems A1.1, A1.2, 1.52, 1.53, 1.54

Problem A1.1

Determine whether or not each of the following continuous-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) $x(t) = 3\cos(\sqrt{2}t + \frac{\pi}{4})$

(b) $x(t) = e^{j(2t-1)}$

(c) $x(t) = [\cos(4t - \frac{\pi}{3})]^2$

(d) $x(t) = \mathcal{O}d\{\cos(4\pi t)u(t)\}$

(e) $x(t) = \mathcal{O}d\{\sin(4\pi t)u(t)\}$

(f) $x(t) = \sum_{n=-\infty}^{\infty} e^{-(3t-n)}u(3t-n)$

(g) $x(t) = \cos(4t) + \sin(4\pi t)$.

Problem A1.2

Determine whether or not each of the following discrete-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) $x[n] = \sin\left(\frac{3}{7}\pi n - 1\right)$

(b) $x[n] = \cos\left(\pi\frac{n}{8} - \pi\right)$

(c) $x[n] = \cos\left(\frac{\pi}{6}n^2\right)$

(d) $x[n] = \cos\left(\frac{\pi}{3}n\right)\cos\left(\frac{\pi}{6}n\right)$

(e) $x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{3}n\right) - 4\cos\left(\frac{1}{2}n + \frac{\pi}{6}\right)$

Problem Set 2 (Due Monday 2/1)

Problems 1.21, 1.22, 1.32, 1.36

Problem Set 3 (Due Monday 2/8)

Problems A1.3, A1.4, 1.31, A1.5

Problem A1.3

Consider the following properties

1. Memoryless
2. Time-invariant
3. Linear
4. Causal
5. Stable

Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answers. In each example, $y(t)$ denotes the system output and $x(t)$ is the system input.

(a) $y(t) = x(t-3) + x(3-t)$

(b) $y(t) = \cos(-3t)x(t)$

(c) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

(d) $y(t) = \begin{cases} 0 & t < 0 \\ x(t) + x(2-t) & t \geq 0 \end{cases}$

(e) $y(t) = \begin{cases} 0 & x(t+2) < 0 \\ x(t) + x(2-t) & x(t+2) \geq 0 \end{cases}$

(f) $y(t) = x(2t)$.

(g) $y(t) = \frac{dx(t-1)}{dt}$

Problem A1.4

Consider the following properties

1. Memoryless
2. Time-invariant
3. Linear
4. Causal
5. Stable

Determine which of these properties hold and which do not hold for each of the following discrete-time systems. Justify your answers. In each example, $y[n]$ denotes the system output and $x[n]$ is the system input.

(a) $y[n] = x[1-n]$

(b) $y[n] = x[2n-2] - 2x[n-8]$

(c) $y[n] = \sin(\pi n)x[n]$

(d) $y[n] = \mathcal{E}v\{x[n+1]\}$

(e) $y[n] = \begin{cases} x[n-1] & n \geq 1 \\ 0 & n = 0 \\ x[n] & n \leq -1 \end{cases}$

(f) $y[n] = \begin{cases} x[n] & n \geq 1 \\ 0 & n = 0 \\ x[n] & n \leq -1 \end{cases}$

(g) $y[n] = x[-4n+1]$

Problem A1.5

Determine if the following systems are

(a) time-invariant

(b) stable

(c) causal

(d) linear.

Justify your answer

1. A system with input $x(t)$ giving output

$$y(t) = \int_{t-10}^{t} \cos(\tau)x(\tau)d\tau$$

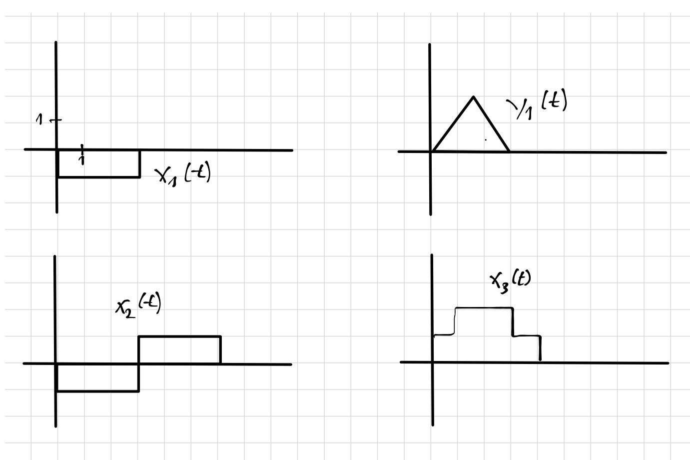
2. A system with input $x[n]$ giving output

$$y[n] = n^2 x[n^2]$$

Problem Set 4 (Due Wednesday 2/17)

Problems 1.29, A1.6, A2.1, 2.24

Problem A1.6



When the input to a linear time-invariant system is given by $x_1(t)$, the output is $y_1(t)$, see the figure.

(a) If the input is $x_2(t)$ (see the figure), what is the output?

(b) If the input is $x_3(t)$ (see the figure), what is the output?

Problem A2.1

Compute the convolution $y[n] = x[n] * h[n]$ of the following pair of signals

(a) $x[n] = \alpha^n u[n-1], h[n] = \beta^n u[n]$, $\alpha \neq \beta$.

(b) $x[n] = \alpha^n u[n-1], h[n] = \alpha^n u[n]$

(c) $x[n] = \left(\frac{1}{2}\right)^n u[n-3], h[n] = (-3)^n u[4-n]$.

(d) $x[n] = \begin{cases} 1 & 0 \leq n \leq 5 \\ 0 & \text{otherwise} \end{cases}$, $h[n] = \begin{cases} 1 & 1 \leq n \leq 5 \\ 1 & 8 \leq n \leq 13 \\ 0 & \text{otherwise} \end{cases}$

Problem Set 5 (Due Monday 2/22)

Problems 2.22, A2.3, A2.4, A2.5

Problem A2.3

The following are the impulse responses of discrete-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers. The justification for stability needs to in terms of the summation condition.

(a) $h[n] = \left(\frac{1}{2}\right)^n u[n]$

(b) $h[n] = (-1.8)^n u[n-2]$

(c) $h[n] = (2)^n u[-n+1]$

(d) $h[n] = \left(\frac{6}{5}\right)^n u[-1-n]$

(e) $h[n] = \left(-\frac{1}{8}\right)^n u[n] + 1.01^{n+1} u[n+1]$

(f) $h[n] = \left(-\frac{1}{8}\right)^n u[n] + 0.99^n u[1-n]$

(g) $h[n] = n^2 \left(\frac{1}{2}\right)^n u[n-3]$

Problem A2.4

The following are the impulse responses of continuous-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers. The justification for stability needs to in terms of the integration condition.

(a) $h(t) = e^{-4t} u(-t-2)$

(b) $h(t) = e^{-\frac{1}{2}t} u(t-2)$

(c) $h(t) = e^{-1.1t} u(t+100)$

(d) $h(t) = e^{-4t} u(t-2)$

(e) $h(t) = e^{-|t|}$

(f) $h(t) = t^2 e^{-t/2} u(t)$

(g) $h(t) = \left(2e^{-t/2} - e^{(10-t)/10}\right) u(t+2)$

(h) $h(t) = \frac{t}{t^2+1}$

Problem A2.5

1. The impulse response of an LTI system is given by

$$h(t) = \cos(t)u(t)$$

   If the input is $x(t) = u(t)$, calculate the output.

2. The impulse response of an LTI system is given by

$$h[n] = (-2)^{|n|}$$

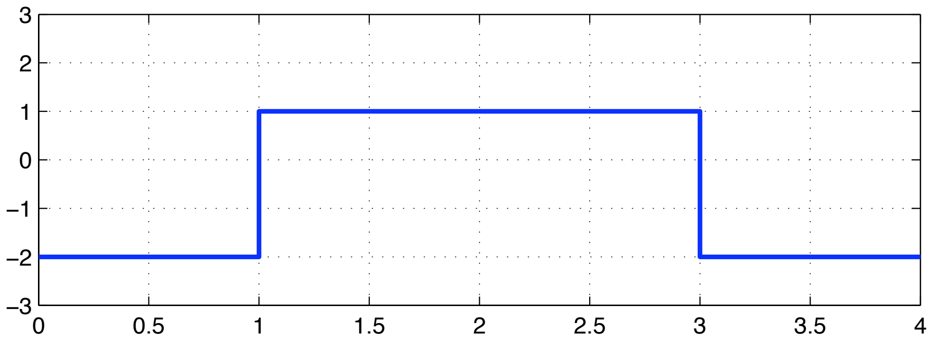
   If the input is $x[n] = u[n] - u[n-2]$, calculate the output.

Problem Set 6 (Due Monday 3/7)

Problems 2.30, 3.22, 3.23, 3.24, 3.26, A3.5

Problem A3.5

The following plot shows a signal $x(t)$ with period $T = 4$.



1. Find the Fourier coefficients of $x(t)$.

2. *Sketch* the the signal with period $T = 4$ having Fourier coefficients

$$a_k = \begin{cases} 0 & k = 0 \\ -3\frac{\sin\left(k\frac{\pi}{2}\right)}{k\pi} & k \neq 0 \end{cases}$$

3. *Sketch* the signal with period $T = 4$ having Fourier coefficients

$$a_k = \begin{cases} -\frac{1}{2} & k = 0 \\ -6e^{j\frac{\pi}{4}k}\frac{\sin\left(k\frac{\pi}{2}\right)}{k\pi} & k \neq 0 \end{cases}$$

4. *Sketch* the signal with period $T = 4$ having Fourier coefficients

$$a_k = \begin{cases} -\frac{1}{2} & k = 0 \\ 3j\frac{\sin\left(k\frac{\pi}{2}\right)}{k^2\pi} & k \neq 0 \end{cases}$$