EE213 Spring 2008
Problem Set 2 Solutions

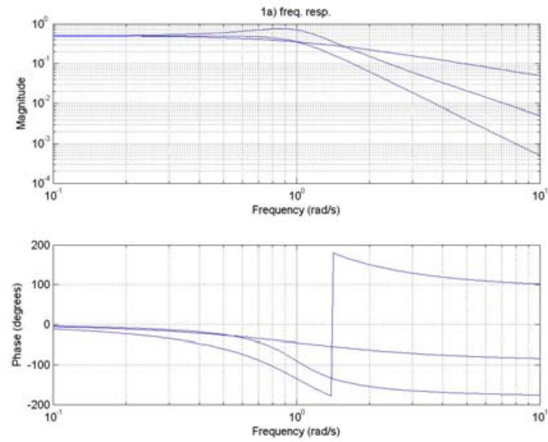
1) For this problem let H_i be transfer function for ith circuit,

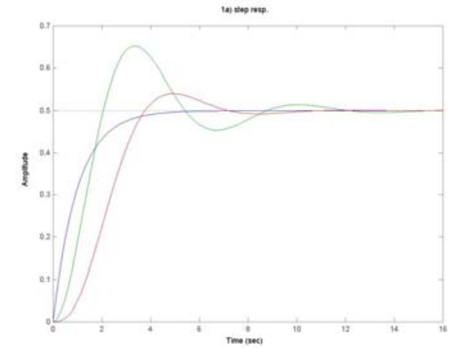
a) See first homework solutions for transfer functions. All filters are lowpass filters with cutoff frequency around $\omega = 1$. Higher order H_i have sharper cutoffs in stopband (where $\omega > 1$). Phase over frequency range for H_i decreases by $i\pi/2$. Step response increases to .5 with higher order filters having a more sluggish response.

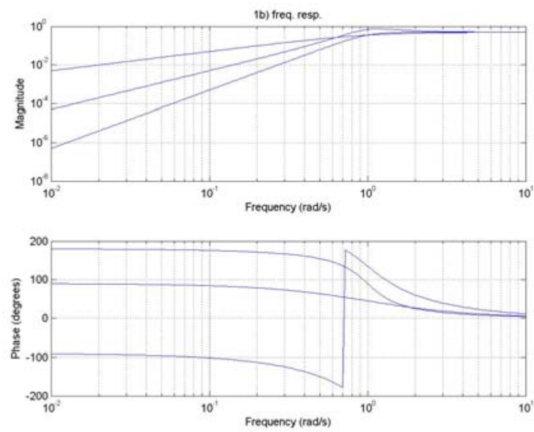
b) For all filters substitute $1/\omega$ instead of ω in part a) to get

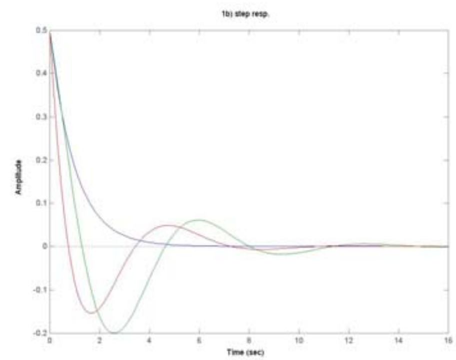
$$H_1 = \frac{j\omega}{2(1+j\omega)}, \quad H_2 = \frac{(j\omega)^2}{2(1+\sqrt{2}(j\omega)+(j\omega)^2)}, \quad H_3 = \frac{(j\omega)^3}{2(1+j\omega)(1+j\omega+(j\omega)^2)}.$$

Here filters are highpass filters with cutoff frequency around $\omega = 1$. Higher order H_i have sharper cutoffs in stopband (where $\omega < 1$). Phase is same as part a), but add $i\pi/2$ to phase for H_i. Step response is a decaying exponential for H_1 with quicker decay for higher order filters.









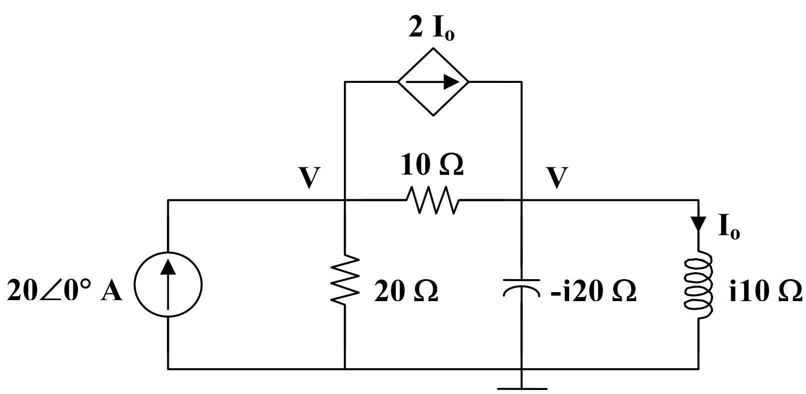
Chapter 10, Solution 12.

$$20\sin(1000t) \longrightarrow 20\angle 0°, \quad \omega = 1000$$

$$10 \text{ mH} \longrightarrow j\omega L = j10$$

$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(50\times 10^{-6})} = \text{-}j20$$

The frequency-domain equivalent circuit is shown below.



At node 1,

$$20 = 2\mathbf{I}_o + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10}, \qquad \text{where}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10}$$

$$20 = \frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10}$$

$$400 = 3\mathbf{V}_1 - (2 + j4)\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10} = \frac{\mathbf{V}_2}{-j20} + \frac{\mathbf{V}_2}{j10}$$

$$j2\mathbf{V}_1 = (-3 + j2)\mathbf{V}_2$$

or

$$\mathbf{V}_1 = (1 + j1.5)\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$400 = (3 + j4.5)\mathbf{V}_2 - (2 + j4)\mathbf{V}_2 = (1 + j0.5)\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{400}{1 + j0.5}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10} = \frac{40}{j(1 + j0.5)} = 35.74\angle -116.6°$$

Therefore, $i_o(t) =$ **35.74 sin(1000t – 116.6°) A**

Matlab commands:

```
A=[3 -(2+j*4);j*2 3-j*2];
b= [400;0];
V=inv(A)*b;
Io=V(2)/(j*10);
```

Chapter 10, Solution 39.

For mesh 1,

$$(28 - j15)I_1 - 8I_2 + j15I_3 = 12\angle 64^o \qquad (1)$$

For mesh 2,

$$-8I_1 + (8 - j9)I_2 - j16I_3 = 0 \qquad (2)$$

For mesh 3,

$$j15I_1 - j16I_2 + (10 + j)I_3 = 0 \qquad (3)$$

In matrix form, (1) to (3) can be cast as

$$\begin{pmatrix} (28 - j15) & -8 & j15 \\ -8 & (8 - j9) & -j16 \\ j15 & -j16 & (10 + j) \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 12\angle 64^o \\ 0 \\ 0 \end{pmatrix} \quad \text{or} \quad AI = B$$

Using MATLAB,
```
A=[28-j*15 -8 j*15;-8 (8-j*9) -j*16;j*15 -j*16 (10+j)];
b=[12*exp(j*pi*64/180);0;0];
I=inv(A)*b;
```

$$I_1 = -0.128 + j0.3593 = \underline{0.3814\angle 109.6^o \text{ A}}$$

$$I_2 = -0.1946 + j0.2841 = \underline{0.3443\angle 124.4^o \text{ A}}$$

$$I_3 = 0.0718 - j0.1265 = \underline{0.1455\angle -60.42^o \text{ A}}$$

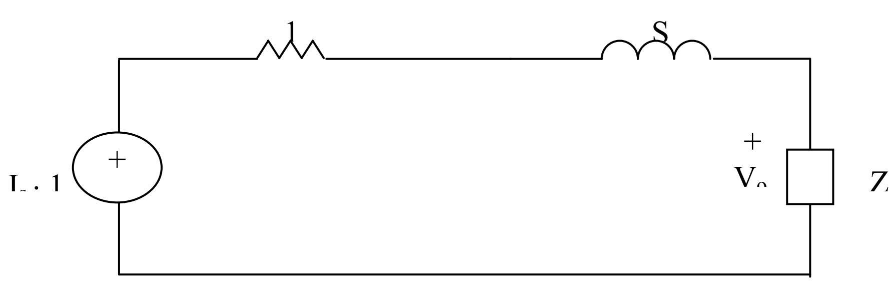
$$I_x = I_1 - I_2 = 0.0666 + j0.0752 = \underline{0.1005\angle 48.5^o \text{ A}}$$

Chapter 14, Solution 6.

$$1H \longrightarrow j\omega L = sL = s$$

Let $Z = s//1 = \dfrac{s}{s+1}$

We convert the current source to a voltage source as shown below.

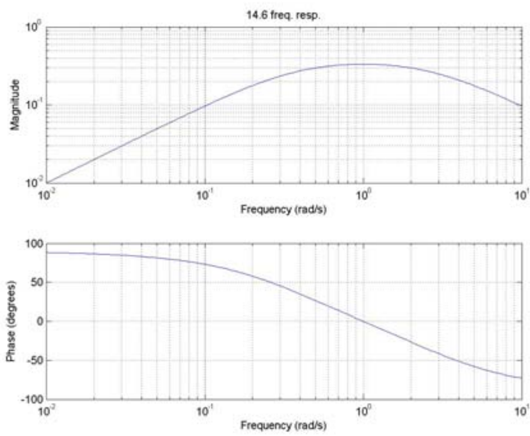


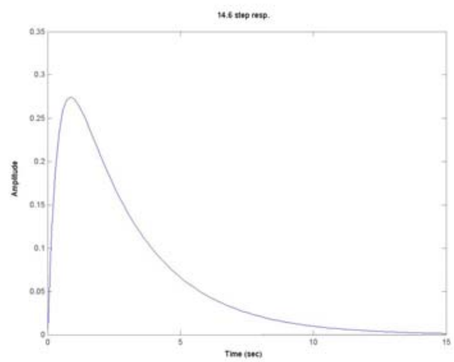
$$V_o = \frac{Z}{Z+s+1}(I_s x1) = \frac{\frac{s}{s+1}}{s+1+\frac{s}{s+1}} I_s = \frac{sI_s}{(s+1)^2+s} = \frac{sI_s}{s^2+3s+1}$$

$$I_o = \frac{V_o}{1} = \frac{sI_s}{s^2+3s+1}$$

$$H(s) = \frac{I_o}{I_s} = \frac{s}{s^2+3s+1}$$

This is a bandpass filter with center frequency around 1rad/s. Phase is similar to first order lowpass filter from 1a). Step response achieves maximum at around 1sec. with an amplitude around .27.





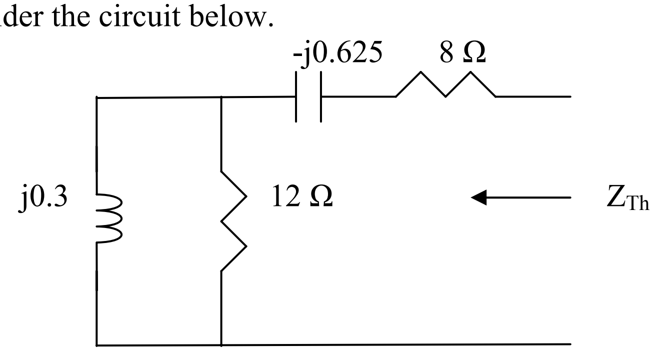
Chapter 11, Solution 14.

We find the Thevenin equivalent at the terminals of Z.

40 mF $\longrightarrow$ $\frac{1}{j\omega C} = \frac{1}{j40x40x10^{-3}} = j0.625$

7.5 mH $\longrightarrow$ $j\omega L = j40x7.5x10^{-3} = j0.3$

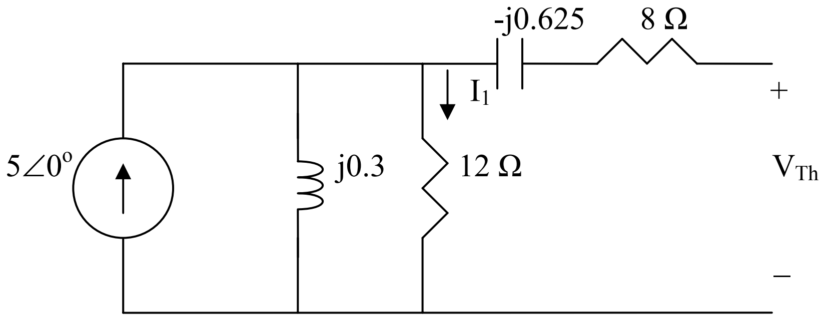
To find Z_{Th}, consider the circuit below.



$$Z_{Th} = 8 - j0.625 + 12 // j0.3 = 8 - j0.625 + \frac{12x0.3}{12+0.3} = 8.0075 - j0.3252$$

$Z_L = (Z_{Thev})^* =$ **8.008 + j0.3252Ω**.

To find V_{Th}, consider the circuit below.



By current division,

$I_1 = 5(j0.3)/(12+j0.3) = 1.5\angle 90°/12.004\angle 1.43° = 0.12496\angle 88.57°$

$= 0.003118 + j0.12492A$

$V_{Thev\ rms} = 12I_1/\sqrt{2} = 1.0603\angle 88.57°V$

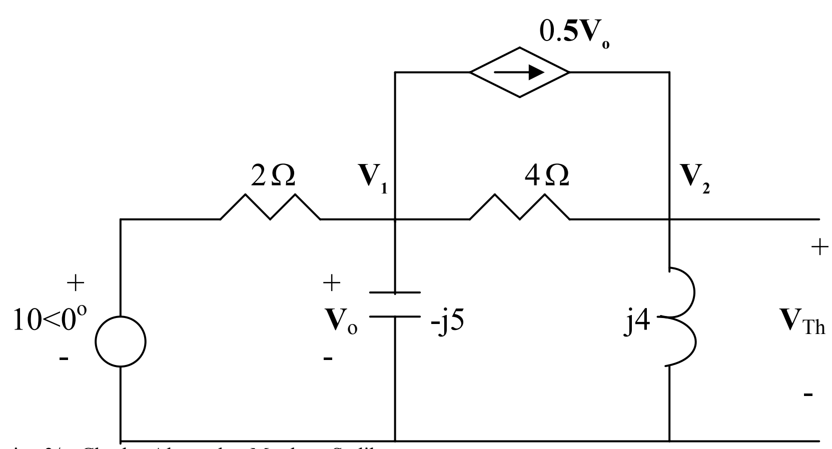
$I_{Lrms} = 1.0603\angle 88.57°/2(8.008) = 66.2\angle 88.57°mA$

$P_{avg} = |I_{Lrms}|^2 8.008 =$ **35.09 mW**.

Chapter 11, Solution 16.

$$\omega = 4, \quad 1H \longrightarrow j\omega L = j4, \quad 1/20F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j4x1/20} = -j5$$

We find the Thevenin equivalent at the terminals of Z_L. To find V_{Th}, we use the circuit shown below.



At node 1,

$$\frac{10-V_1}{2} = \frac{V_1}{-j5} + 0.5V_1 + \frac{V_1 - V_2}{4} \longrightarrow 5 = V_1(1.25 + j0.2) - 0.25V_2 \qquad (1)$$

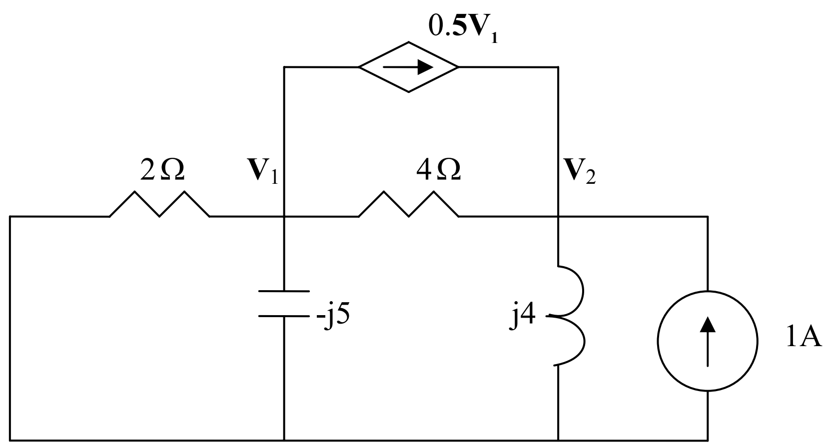
At node 2,

$$\frac{V_1 - V_2}{4} + 0.25V_1 = \frac{V_2}{j4} \longrightarrow 0 = 0.5V_1 + V_2(-0.25 + j0.25) \qquad (2)$$

Solving (1) and (2) leads to

$$V_{Th} = V_2 = 6.1947 + j7.0796 = 9.4072\angle 48.81^o$$

To obtain R_{Th}, consider the circuit shown below. We replace Z_L by a 1-A current source.



At node 1,

$$\frac{V_1}{2} + \frac{V_1}{-j5} + 0.25V_1 + \frac{V_1 - V_2}{4} = 0 \longrightarrow 0 = V_1(1 + j0.2) - 0.25V_2$$

(3)

At node 2,

$$1 + \frac{V_1 - V_2}{4} + 0.25V_1 = \frac{V_2}{j4} \longrightarrow -1 = 0.5V_1 + V_2(-0.25 + j0.25) \qquad (4)$$

Solving (1) and (2) gives

$$Z_{Th} = \frac{V_2}{1} = 1.9115 + j3.3274 = 3.8374\angle 60.12^o$$

$$P_{\max} = \frac{|V_{Th}|^2}{8R_{Th}} = \frac{9.4072^2}{8x1.9115} = \underline{5.787\text{ W}}$$