1) (a)

```matlab
>> syms H k s H2a H2b t impulse2a impulse2b step2a step2b out2a out2b
>> H = k / (1 + s^2 + (3-k)*s)
>> H2a=subs(H,k,3);[n2a,d2a]=numden(H2a);p2a=solve(d2a,s),
   <p2a=sqrt(-1)
   -sqrt(-1)>
>> H2a=3/(s-p2a(1))/(s-p2a(2));
   step2a='heaviside(t+abs(eps))'*ilaplace(H2a/s,s,t),
   <step2a=heaviside(t+abs(eps))*(-3*cos(t)+3)>
>> impulse2a='heaviside(t+abs(eps))'*ilaplace(H2a,s,t),
   <impulse2a=3*heaviside(t+abs(eps))*sin(t)>
>> IN=laplace('heaviside(t)*heaviside(-t+1)',t,s);
   out2a='heaviside(t+abs(eps))'*ilaplace(IN*H2a,s,t)
   <out2a=heaviside(t+abs(eps))*((3*cos(t-1)-3)*
   heaviside(t-1)-3*cos(t)+3)>
>> subplot(131),ezplot(impulse2a,-1,20),ylabel('impulse(t)'),title('k=3')
>> subplot(132),ezplot(step2a,-1,20),ylabel('step(t)'),title('k=3')
>> subplot(133),ezplot(out2a,-1,20),ylabel('out(t)'),title('k=3')
```

Each response of this circuit contains a sum of terms. The terms from the circuit are of the form sin(t+a)u(t) only the weighting, phase, and/or delay of these terms change as the circuit input changes. The contributions from the circuit are the result of the two complex conjugate poles at +i and -i. When the input contains a unit step or the delay of a
Each response of this circuit contains a sum of terms. The terms from the circuit are of the form \(e^{t/2}\sin(3^{1/2}t/2+a)u(t)\) only the weighting, phase, and/or delay of these terms change as the circuit input changes. These exploding contributions from the circuit are the result of the two complex conjugate poles at \((1+i3^{1/2})^2\) and \((1+i3^{1/2})^2\) in the right half plane. When the input contains a unit step or the delay of a unit step, these same terms appropriately weighted also appear in the output.
c) The response of circuit to a particular input always contains terms that arise from the input and from the circuit. The responses from the circuit are primarily determined by the poles of the circuit; only the weighting of the terms and the phases of sinusoids are influenced by the input and zeros of the circuit. When \( k < 2 \) we have two distinct real poles in left half plane. When \( k = 2 \) we have two repeated poles at \(-1\). When \( 2 < k < 3 \) we have two complex poles in the left half plane. When \( k = 3 \) there are two poles on the jw axis. When \( 3 < k < 5 \) there are two complex poles in the right half plane. When \( k > 5 \) there are two real poles in the right half plane. As \( k \) grows large one pole will be close to zero in the right half plane and the second pole will be close to \( 2k \). All real poles in the left half plane contribute one-sided decaying exponentials; the time constants of these poles are the reciprocal of the magnitude of the pole; poles further from the imaginary axis decay more quickly. All real poles in the right half plane contribute one-sided exploding exponentials; the time constants of these poles are the reciprocal of the magnitude of the pole; poles further from the imaginary axis increase more quickly. All complex poles in the left half plane contribute one-sided decaying exponentials times sinusoids; the time constants of these poles are the reciprocal of the magnitude of the pole; poles further from the imaginary axis decay more quickly. The frequencies of the sinusoids are the imaginary part of the complex poles. All complex poles in the right half plane contribute one-sided exploding exponentials times sinusoids; the time constants of these poles are the reciprocal of the magnitude of the real part of the pole; poles further from the imaginary axis explode more quickly. The frequencies of the sinusoids are the imaginary part of the complex poles.

2) a) syms A b in out s Ha
\[ A = \begin{bmatrix} 2+s & -1 & 0 \\ 1+s & -s & -1 \\ 2 & 0 & -1 \end{bmatrix} \]
\[ b = [s;0;0] \]
\[ Ha = [0 0 1]*\text{inv}(A)*b \]
\[ Ha = \frac{2s^2}{s+s^2+1} \]

syms impulse step sinu t

impulse = 'heaviside(t+abs(eps))*ilaplace(Ha)'
impulse =

\[ \text{heaviside}(t+\text{abs}(eps))*\{2*\text{dirac}(t)-2*\exp(-1/2*t)*\cos(1/2*3^{(1/2)}*t) - 2/3*3^{(1/2)}*\exp(-1/2*t)*\sin(1/2*3^{(1/2)}*t)\} \]
step = 'heaviside(t+abs(eps))*ilaplace(Ha/s)'
step =
heaviside(t+abs(eps))*(2*exp(-1/2*t)*cos(1/2*3^(1/2)*t)-2/3*3^(1/2)*exp(-1/2*t)*sin(1/2*3^(1/2)*t))

sinu = 'heaviside(t+abs(eps))*ilaplace(Ha*laplace(cos(t)*heaviside(t)))'
sinu =
heaviside(t+abs(eps))*(-2*sin(t)+2*exp(-1/2*t)*cos(1/2*3^(1/2)*t)+2/3*3^(1/2)*exp(-1/2*t)*sin(1/2*3^(1/2)*t))

subplot(311); ezplot(impulse); ylabel('impulse resp.');
subplot(312); ezplot(step); ylabel('step resp.);
subplot(313); ezplot(sinu); ylabel('sinusoid. resp.);

Transfer function represents a second order highpass filter.

b) syms Hb
A = [2+1/s -1; -1 2+s]
b = [1;0]
Hb=[0 1]*inv(A)*b

impulse = 'heaviside(t+abs(eps))*ilaplace(Hb)'
impulse =
heaviside(t+abs(eps))*(1/2-1/2*t)*exp(-t)
step = 'heaviside(t+abs(eps))'*ilaplace(Hb/s)
step =
1/2*heaviside(t+abs(eps))*t*exp(-t)

sinu = 'heaviside(t+abs(eps))'*ilaplace(Hb*laplace(cos(t)*heaviside(t)))
sinu =
heaviside(t+abs(eps))*(1/4*cos(t)+(1/4*t-1/4)*exp(-t))

subplot(311); ezplot(impulse); ylabel('impulse resp.'); axis([0 5 -.1 .5])
subplot(312); ezplot(step); ylabel('step resp.'); subplot(313); ezplot(sinu); ylabel('sinusoid. resp.);

Transfer function represents a second order bandpass filter.

3) Chapter 15, Solution 43.

(a) For 0 < t < 1, x(t - λ) and h(λ) overlap as shown in Fig. (a).

\[ y(t) = x(t) * h(t) = \int_{0}^{1} (1)(\lambda) \, d\lambda = \frac{\lambda^2}{2} \bigg|_{0}^{1} = \frac{1}{2} \]
For \( 1 < t < 2 \), \( x(t - \lambda) \) and \( h(\lambda) \) overlap as shown in Fig. (b).

\[
y(t) = \int_{\lambda_1}^{\lambda_2} (1(\lambda)) \, d\lambda + \int_{\lambda_1}^{\lambda_2} (1(\lambda)) \, d\lambda = \frac{\lambda^2}{2} \bigg|_{\lambda_1}^{\lambda_2} + \lambda \bigg|_{\lambda_1}^{\lambda_2} = \frac{1}{2} t^2 + 2t - 1
\]

For \( t > 2 \), there is a complete overlap so that

\[
y(t) = \int_{\lambda_1}^{\lambda_2} (1(\lambda)) \, d\lambda = \lambda \bigg|_{\lambda_1}^{\lambda_2} = t - (t - 1) = 1
\]

Therefore,

\[
y(t) = \begin{cases} 
  \frac{t^2}{2}, & 0 < t < 1 \\
  -(t^2/2) + 2t - 1, & 1 < t < 2 \\
  1, & t > 2 \\
  0, & \text{otherwise}
\end{cases}
\]

(b) For \( t > 0 \), the two functions overlap as shown in Fig. (c).

\[
y(t) = x(t) * h(t) = \int_0^t 2 e^{-\lambda} \, d\lambda = -2 e^{-\lambda} \bigg|_0^t
\]

Therefore,

\[
y(t) = 2(1 - e^{-t}), \quad t > 0
\]

(c) For \( -1 < t < 0 \), \( x(t - \lambda) \) and \( h(\lambda) \) overlap as shown in Fig. (d).

\[
y(t) = x(t) * h(t) = \int_0^{t+1} (1(\lambda)) \, d\lambda = \frac{\lambda^2}{2} \bigg|_{t}^{t+1} = \frac{1}{2} (t + 1)^2
\]
For $0 < t < 1$, $x(t - \lambda)$ and $h(\lambda)$ overlap as shown in Fig. (e).

\[ y(t) = \int_0^t (1)(\lambda) \, d\lambda + \int_{t+1}^{t+2} (1)(2 - \lambda) \, d\lambda. \]

\[ y(t) = \frac{\lambda^2}{2} \bigg|_0^1 + \left(2\lambda - \frac{\lambda^2}{2}\right) \bigg|_{t+1}^{t+2} = -\frac{1}{2} t^2 + t + \frac{1}{2} \]

For $1 < t < 2$, $x(t - \lambda)$ and $h(\lambda)$ overlap as shown in Fig. (f).

\[ y(t) = \int_{t-1}^0 (1)(\lambda) \, d\lambda + \int_t^{t+1} (1)(2 - \lambda) \, d\lambda. \]

\[ y(t) = \frac{\lambda^2}{2} \bigg|_{t-1}^0 + \left(2\lambda - \frac{\lambda^2}{2}\right) \bigg|_t^{t+1} = -\frac{1}{2} t^2 + t + \frac{1}{2} \]

For $2 < t < 3$, $x(t - \lambda)$ and $h(\lambda)$ overlap as shown in Fig. (g).

\[ y(t) = \int_{t-1}^t (1)(2 - \lambda) \, d\lambda = \left(2\lambda - \frac{\lambda^2}{2}\right) \bigg|_{t-1}^t = \frac{9}{2} - 3t + \frac{1}{2} t^2 \]
Therefore,
\[
y(t) = \begin{cases} 
(t^2/2) + t + 1/2, & -1 < t < 0 \\
-(t^2/2) + t + 1/2, & 0 < t < 2 \\
(t^2/2) - 3t + 9/2, & 2 < t < 3 \\
0, & \text{otherwise}
\end{cases}
\]

Chapter 15, Solution 46.

(a) \[x(t) * y(t) = 2\delta(t) * 4u(t) = 8u(t)\]

(b) \[x(t) * z(t) = 2\delta(t) * e^{-2t}u(t) = 2e^{-2t}u(t)\]

(c) \[y(t) * z(t) = 4u(t) * e^{-2t}u(t) = \int_0^t e^{-2\lambda} d\lambda = \left[\frac{4e^{-2\lambda}}{-2}\right]_0^t = 2(1-e^{-2t})\]

(d) \[y(t) *[y(t) + z(t)] = 4u(t) *[4u(t) + e^{-2t}u(t)] = 4\int[4u(\lambda) + e^{-2\lambda}u(\lambda)]d\lambda
\]
\[= 4\int[4 + e^{-2\lambda}]d\lambda = 4\left[4t + \frac{e^{-2\lambda}}{-2}\right]_0^t = 16t - 2e^{-2t} + 2\]

Matlab code

```matlab
% Matlab code

t=-2:.01:5;
x1=heaviside(t+abs(eps)) - heaviside(t-1+abs(eps));
h1=heaviside(t+abs(eps)) -(1-t).*heaviside(t+abs(eps)).*heaviside(1-t-abs(eps));
y1=conv(x1,h1)*.01;

x2=heaviside(t+abs(eps));
h2=2*exp(-t).*heaviside(t+abs(eps));
y2=conv(x2,h2)*.01;

x3=heaviside(t+1+abs(eps)).*heaviside(1-t-abs(eps));
h3=t.*(heaviside(t+abs(eps)) -heaviside(t-1+abs(eps)))+ (2-t).*(heaviside(t-1+abs(eps))-heaviside(t-2+abs(eps)));
y3=conv(x3,h3)*.01;

t0=-4:.01:5;
subplot(3,1,1); plot(t0,y1);ylabel('16.43a');axis([-1 5 0 1.2]);
subplot(3,1,2); plot(t0,y2);ylabel('16.43b');axis([-1 5 0 2.2]);
subplot(3,1,3); plot(t0,y3);ylabel('16.43c');axis([-1 5 0 1.2]);
```
syms t tau x1 x10 x2 x3 h1 h2 h3 h4 y1 y10 y2 y3 y4
x1='heaviside(t-1-tau-abs(eps))'
x10='heaviside(t-tau-abs(eps))'
h1='heaviside(tau+abs(eps))'-\( (1-\tau) \cdot \text{heaviside}(\tau+\abs{\eps}) \cdot \text{heaviside}(1-\tau-\abs{\eps}) \);
y1=int(x1*h1,tau,-inf,inf)
y10=int(x10*h1,tau,-inf,inf)
y1=y10-y1
h2=exp(-tau)*'heaviside(tau+abs(eps))'\)
y2=int(x10*h2,tau,-inf,inf)
x3='heaviside(t+1-tau-abs(eps))'
h3=tau*'heaviside(tau+abs(eps))'*'heaviside(1-tau-abs(eps))'
h4=(2-tau)*'heaviside(tau-1+abs(eps))'*'heaviside(2-tau-abs(eps))'
y3=int(x3*(h3+h4),tau,-inf,inf)
y4=int(x1*(h3+h4),tau,-inf,inf)
y3=y3-y4
t=-1:.01:5;
x=0*t;x(101)=200;
y=heaviside(t+abs(eps));
z=exp(-2*t).*y;
a=conv(x,y)*.01;
b=conv(x,z)*.01;
c=conv(y,z)*.01;
d=conv(y,y+z)*.01;
t0=-2:.01:10;

subplot(411);plot(t0,a);ylabel('16.46a)');axis([-1 5 0 2.2]);
subplot(412);plot(t0,b);ylabel('16.46b)');axis([-1 5 0 2.2]);
subplot(413);plot(t0,c);ylabel('16.46c)');axis([-1 5 0 1.2]);
subplot(414);plot(t0,d);ylabel('16.46d)');axis([-1 5 0 6.2]);

syms x y y2 z t tau a b c d
x=dirac(t-tau);
y='heaviside(tau+abs(eps))';
z=exp(-tau)*y;

a=int(x*y,tau,-inf,inf)
b=int(x*z,tau,-inf,inf)
y2='heaviside(t-tau-abs(eps))';
c=int(y2*z,tau,-inf,inf)
d=int(y2*(y+z),tau,-inf,inf)