EE 342 Final
May 11, 2007
Closed Book, 3 crib sheets, Justify all work
Good Luck

NAME__________________________

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1) (15) A satellite operates in three modes of operation; it is in green mode 80% of the time, it in yellow mode 15% of the time, and in red mode 5% of the time. In green mode there is a .1 probability of software failure, in yellow mode there is a .2 probability of failure, and in red mode there is a .5 probability of failure.

a) Find the probability of software failure.

b) Find the probability that computer was in yellow mode given a software failure.

c) Let the probability of hardware failure be .2. If hardware failures occur independently of software failures, find the probability that the satellite fails (hardware or software failure occurs).
2) (12) You have 1000 free minutes per month on your cellular phone. Assume the duration of all calls are independent and identically distributed and have the following pmf in minutes,

\[ p_X(k) = (0.25)(0.75)^{k-1}, \ k = 1, 2, \ldots \]

What is the maximum number of calls you can you make such that there is a probability of .99 that you will not go over your allotted free minutes in one month. Use Central Limit Theorem approximations.
3) (18) The input to a communication channel is denoted by the random variable $X$ and the output by the random variable $Y$. $X$ has pmf

$$p_X(x) = .4\delta(x - 1) + .3\delta(x) + .3\delta(x + 1).$$

The conditional pmf of $Y$ given $X$ is

$$p_{Y|X}(y|1) = .8\delta(y - 1) + .2\delta(y)$$

$$p_{Y|X}(y|0) = .6\delta(y) + .2\delta(y - 1) + .2\delta(y + 1)$$

$$p_{Y|X}(y|1) = .8\delta(y + 1) + .2\delta(y).$$

a) Find the joint pmf of $X$ and $Y$.

b) Find the pmf of $Y$.

c) Find the conditional probability, $P(X = 0|Y = 0)$. 
4) (28) Let $X$ and $Y$ be iid uniform random variables defined on the interval $[0, 1]$.

a) Find the pdf of $Z = XY$.

b) Find the pdf of $W = \max(X, Y)$.

c) Write matlab code to simulate 1000 samples drawn from $Z$ and $W$. 
5) (36) Let $X$ and $Y$ be random variables with joint pdf given by

$$f_{X,Y}(x, y) = \begin{cases} 
2 \exp(-x - y), & \text{if } 0 \leq x \leq y < \infty \\
0, & \text{otherwise.}
\end{cases}$$

a) Find the marginal pdfs of $X$ and $Y$. Find the conditional pdf, $f_{X|Y}(x|y)$.

b) Find all first and second order statistics of $X$ and $Y$: $E(X)$, $E(Y)$, $\text{Var}(X)$, $\text{Var}(Y)$, and $\text{Cov}(X,Y)$.

c) Find $\hat{X}_{MMSE}$ and the resulting MSE. (This is the no observation case.)

d) Find $\hat{X}(Y)_{LMSE}$ and the resulting MSE.

e) Find $\hat{X}(Y)_{MMSE}$. 

6) (25) Let $X$ and $Y$ have the following moment generating function.

$$M_{X,Y}(t, s) = \exp(t - s + 2t^2 + 4.5s^2 - 3ts)$$

a) Find the joint pdf of $X$ and $Y$.

b) Use matlab commands to simulate 1000 draws of $X$ and $Y$.

c) Find $E(Y|X)$, $VAR(X + 2Y)$, and $E(X^3)$. 
7) (26) Each part is independent of other parts.

a) Let the variance of $X$ equal 4 and the variance of $Y$ equal 25. If the correlation coefficient between $X$ and $Y$ is $\rho = -0.2$ find the $\text{COV}(X, Y)$. If $Z = X - 2Y$ find the variance of $Z$.

b) Let $X$ have the following pmf

$$p_X(k) = P(X = k) = \frac{(\lambda)^k}{k!} e^{-\lambda} \quad k = 0, 1, \ldots, \lambda > 0$$

and $Y$ have the following pmf

$$p_Y(k) = P(Y = k) = \frac{(\mu)^k}{k!} e^{-\mu} \quad k = 0, 1, \ldots, \mu > 0.$$  

Assume $X$ and $Y$ are independent. Let $Z = X + Y$. Find the pmf of $Z$. (Hint: use moment generating functions.)

c) An urn contains ten balls; five balls are labeled one and five balls are labeled zero. Six balls are picked at random without replacement. Let $X$ denote the sum of the labels of these six balls. Find the mean and variance of $X$. 